Persistent Inequality, Corruption, and Factor Productivity

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Abstract

I build a model with bequests, financial frictions and corrupt bureaucrats to explain the link between corruption and inequality and its effects on productivity. Because of collateral requirements, profits are determined by wealth. If individual wealth is not publicly observed, taxation is regressive under corruption. When wealth inequality is high, corruption is more prevalent, creating persistent feedback between corruption and inequality. I calibrate the model and investigate the effect of corruption on inequality and TFP. Through regressive taxation, corruption induces wealth levels to inversely affect the productivity selection. This in turn has adverse effects on aggregate TFP.

JEL classifications: E02; E24; H2; O4
Keywords: inequality, corruption, financial frictions, productivity, size distribution.

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1 Introduction

For a large fraction of the world’s population, interactions with the government can be cumbersome, especially for individuals with a limited amount of funds. In their transactions with the state, individuals in corrupt countries do not always have equal access to government services. Entrepreneurs connected with government officials, typically as a result of some form of illicit payment, tend to see real economic returns. If wealth is an important factor (directly or indirectly) in dealings with corrupt government officials, such interactions may play a role in exacerbating inequality and its effects. Figure 1 illustrates the positive correlation between income inequality and corruption.

![Figure 1: Source, WB Governance Index and UNU-WIID.](image)

The link between inequality and corruption raises a series of questions. What are the mechanisms that translate inequality into corruption, and vice versa? What effects, if any, do such mechanisms have on aggregate productivity and the size distribution of firms? What are the quantitative predictions on the aggregate variables involved? To answer these questions I construct a dynastic model with bequests, in which raising capital is hampered by collateral requirements (financial frictions). The government raises revenue through taxes on entrepreneurial profits, which affect entry into entrepreneurship. Corrupt tax collectors can be bribed to lower taxes on profit and when wealth is private information,

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1 See the World Bank’s *Doing Business Report.*
3 For the empirical evidence see You and Khagram (2004) and Gupta et. al. (1998).
wealthier entrepreneurs pay lower taxes (*de-facto* regressive taxation). Because taxes act as an entry barrier, wealthier individuals face lower entry thresholds as measured by the productivity level that makes them indifferent between renting out their wealth or using it as entrepreneurial capital. An outcome of the private information model is that, under certain specifications, higher inequality implies a higher frequency of corrupt transactions. In this manner the model can explain some of the persistence in both inequality and corruption and provide a link between inequality and productivity outcomes. The quantitative results suggest that corruption can account for a reallocation of up to an additional 7% of aggregate income to the top 1% of the income distribution. It also suggests that the middle of the size distribution of firms is especially thin in more corrupt countries and the lower tail is significantly thicker.

The mechanisms at work can be decomposed in the following steps. First, because of financial frictions, one’s wealth holdings are an important determinant of entrepreneurial income. Second, when wealth holdings are private information, corrupt bureaucrats screen individuals by offering a menu of choices that is composed of an assigned tax rate and the bribe the official asks for it. Individuals then self select by choosing the optimal menu of bribes and tax rates. This menu offers lower tax rates to wealthier individuals because they are, on the margin, more sensitive to taxation and therefore willing to pay heftier bribes. In turn, taxation becomes regressive under corrupt regimes, which increases inequality. Third, because taxation acts as a barrier to entry, and because wealthier individuals face lower tax rates, they face lower entry thresholds, which implies that on average wealthy individuals are more likely to become entrepreneurs. In this way corruption increases inequality through both channels; regressive taxation and the choice of whether to become an entrepreneur. Fourth, when the distribution of wealth is particularly unequal, bureaucrats have very noisy ‘information’ about where a particular individual stands on the wealth distribution. To optimize, they increase the fraction of the population from which they accept bribes. This increases the number of corrupt transactions and lowers the fees that the upper tail of the wealth distribution has to pay, implicitly allowing wealthier types to keep a larger proportion of their income not only through lower taxation but also through lower bribes. As a result inequality increases, which feeds back to corruption in a persistent loop. Fifth, because of bequests inequality is transmitted across generations, which

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4On the empirical evidence for this fact see Hunt and Laszlo (2008).
induces persistence.

As a check on the validity of the model I study the effect of corruption on the size distribution of firms measured by both effective labour as well as productivity. In the absence of corruption, there is a unique entry threshold on productivity that is determined by market aggregates and taxes. When corruption is present, bureaucrats screen individuals on their wealth by offering contracts that reduce taxes as wealth increases. Some individuals are excluded from these contracts because of their position in the wealth distribution. Those included in the contract have lower entry (productivity) thresholds, and some enter production where they would not have in the absence of bribery. This in effect creates two productivity thresholds; one that is lower than the unique threshold faced by all in the absence of bribery, and a higher threshold faced only by those who are positioned low enough on the wealth distribution to be excluded from paying bribes. Therefore, a large proportion of entrepreneurs in the middle of the productivity distribution do not find it worthwhile to enter production because of their wealth holdings, resulting in the contraction of the middle of the distribution. Bearing in mind the previously mentioned feedback effects of inequality on corruption, we see that inequality also has adverse effects on the productivity distribution of operating firms.

The financial frictions component is essential for these outcomes for two reasons; first it is a factually relevant tool that links wealth with income thus making the wealth transmission mechanism pertinent for the transmission of inequality (on this point see Banerjee and Newman (1993), Cagetti and De Nardi (2006), Buera (2008, 2009), Buera and Shin (2010), Moll (2009, 2010) for a representative sample). Second it has direct effects on the mechanism through which corruption affects inequality. As the quantitative section demonstrates, when financial frictions are particularly severe, the inequality and productivity distortions discussed above are more pronounced. This is because entry thresholds depend on taxation through financial frictions. When collateral requirements are high, profits depend heavily on wealth levels and taxation can significantly lower entry incentives. On the other hand, when collateral requirements are low, the effect of taxation is minimal since capital is less dependent on wealth holdings. In this manner financial frictions compound the negative effects on inequality and the size distribution of firms that result from a corrupt bureaucracy.

The quantitative section is based on the effects that corruption has on outcomes for a
benchmark ‘clean’ economy. I chose Sweden for the benchmark economy as it is the second least corrupt country in the World Bank Governance Index. I then increase the level of corruption and document its effect on the steady state measures of the GINI coefficient, the percentage of aggregate income that accrues to the top 1% of the distribution, and productivity. I vary the financial frictions coefficient to quantify the importance of the channel for the above mentioned outcomes. Through the mechanisms outlined above, corruption has a significant effect on inequality, whether measured by the percentage of income going to the top percentile of the distribution, or by the GINI coefficient.

Previous attempts to explain the feedback link between corruption and inequality include Alesina and Angeletos (2005). In their specification, the government’s goal is to reduce inequality and it does so by redistributing income. If voters are sufficiently concerned with fairness, larger inequality leads to demands for more redistribution. When corruption is prevalent, redistribution ends up favouring the rich, which increases inequality and restarts the loop. It is difficult however to reconcile this account with the fact that most corrupt governments have very low revenue raising capacity (see for example Besley and Persson 2011) and are therefore unable to redistribute income that they are incapable of collecting. This paper contends that while the channel that links fairness to inequality may be present, the distortionary effect of collecting revenue is the main culprit here. On the other hand Esteban and Ray (2006) caution us that even in cases where officials are honest, lobbying can distort the signals they receive, thus causing inefficient allocation of resources. Environments with high levels of inequality can then increase lobbying intensity and amplify inefficiencies. This paper departs from Esteban and Ray along two significant dimensions (among many others); first, in this paper inefficiencies arise because of the way governments extract resources from entrepreneurs, not because of the way resources are allocated. This is an important distinction, because here corruption is essential in understanding how inequality persists whereas, as Esteban and Ray clearly demonstrate, this may not be so in the allocation of resources channel. Second, lobbying as Esteban and Ray define it, is a way to distort the signals government officials get, thereby distorting information about the true state of nature. In this paper corruption is a direct way individuals employ to change how the rules apply to them, thus resulting in their unfair application.

This paper is related to a large body of literature along two dimensions. The first links corruption to macroeconomic outcomes, (see Mauro (1995, 1998), Alesina and Angeletos
(2005), Ehrilch and Lui (1999), Murphy et. al. (1993) among others). To my knowledge, this paper is the first to provide a mechanism through which corruption and inequality are inextricably linked without appealing to government expenditure. It also provides an explicit mechanism that links corruption to factor productivity, a channel that has not received sufficient attention. The second strand of literature studies the effect of inequality on growth and development (see Galor and Zeira (1993), Galor and Moav (2004), Persson and Tabellini (1994), Alesina and Rodrik (1994), Galor Zeira and Vollrath (2008) for a representative sample). This paper’s contribution is to make explicit the distortions that inequality has on the way governments raise taxes and thus exacerbating the persistence of inequality. In this dimension the paper is more in the spirit of Alesina and Rodrik (1994), where wealth distributions can affect the way individuals interact with the government. I also provide a quantitative framework for analyzing the effects of such frictions (in this case corruption and financial frictions) and do so in a dynamic setting.

For any entrepreneur, interactions with the government are multi-dimensional. They include but are not limited to; taxation, import quotas, and the right to bid on government funded projects. All of these interactions require approval from some government agency and are open to undue influence. This paper reduces all interactions with the government to the single dimension of taxation in order to keep the analysis tractable. Since taxation is used as a convenient proxy for all interactions with the government, it is inevitable that such analysis will underestimate the aggregate effects of corruption.

2 A model of Inequality, Corruption and Productivity

2.1 The Model Environment

Consider a small open dynastic economy populated by a continuum of agents of mass one. Each generation lives for only one period during which it works, produces and consumes. At the end of the period, a new generation of equal mass is born and the old generation dies. At time $t$ each agent is endowed with a productivity level $z_t \in [z, \bar{z}]$ which is distributed according to $F(\cdot)$. Productivity is $iid$ over time. At $t = 0$, the initial old are endowed with wealth distributed according to $H_0(\cdot)$ with support $[\underline{a}_0, \bar{a}_0]$. I will assume that wealth

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5 Productivity is individual specific and can be thought of as the ability to run the project as well as the quality of the project itself. In this case productivity is being used as a generic term to capture the intangibles of the individual production function.
and productivity are uncorrelated.\footnote{This assumption is not crucial to the qualitative outcomes of the model as long as the two are not positively perfectly correlated. In the long run, productivity realizations directly determine wealth and income outcomes, so the model is realistic in that regard.}

Each agent also owns a production technology \( f(k, l) = z k^\alpha l^{1-\alpha} \) where \( \alpha \in (0, 1) \), \( l \) represents labour and \( k \) capital. The agent rents capital at the world rate \( r \) and hires labour at wage \( w \). There are limits to how much capital an individual can borrow. This limit is determined by the amount of collateral an individual can offer. More specifically, \( k \leq \lambda a \) where \( \lambda \geq 1 \) and the amount of collateral is an agent’s wealth endowment. The parameter \( \lambda \) is commonly referred to in the literature as the measure of financial frictions and is a convenient way to parametrize the degree of financial market development.\footnote{See for example Kehoe and Levine (1993).}

Each individual has a unit of time that they supply inelastically. During its lifetime, each generation earns the market determined wage \( w \), and, depending on whether an individual decides to become an entrepreneur, profit \( \pi(a, z) = z k^\alpha l^{1-\alpha} - w l - r(k - a) \). Preferences are given by \( U(c, s) = c^\gamma s^{1-\gamma} \) where \( c \) represents consumption and \( s \) the amount of wealth left as bequests to the next generation. The above utility function implies the following equations for consumption and bequests:

\[
\begin{align*}
c(a, z) &= \gamma (y(z, a) + a) \\
s(a, z) &= (1 - \gamma)(y(z, a) + a) \\
U(a, z) &= \delta (y(z, a) + a)
\end{align*}
\]

where \( y(z, a) \in \{\pi(a, z) + w, w + ra\} \) and \( \delta \equiv \gamma \gamma (1 - \gamma)^{1-\gamma} \). An individual’s profit maximization problem can be written as:

\[
\max_{k,l} \quad z k^\alpha l^{1-\alpha} - w l - r(k - a) \\
s.t \quad k \leq \lambda a
\]

The optimization problem above implies the following individual labour and capital demand functions:

\[
\{k(a, z), l(a, z)\} = \begin{cases} \\
\{\lambda a, \left(\frac{w}{r} \right)^{\frac{1}{1-\alpha}} z^{1/\alpha} \lambda a\} & \text{for } z \geq z_{\min} \\
\{0, 0\} & \text{for } z < z_{\min}
\end{cases}
\]
where \( z_{\text{min}} = \left( \frac{z}{\eta} \right)^{\frac{1}{\alpha}} \) and \( \eta = \alpha \left( \frac{1-\alpha}{w} \right) \). Note that the productivity threshold is the result of the entry decision, which is made on the basis of \( U^e \geq U^w \) where \( U^e = \delta(\pi(a, z) + a + w) \) and \( U^w = \delta(w + (1 + r)a) \) represent the utility of the entrepreneur and worker respectively. The above solution implies the following profits for an active entrepreneur \( (z \geq z_{\text{min}}) \):

\[
\pi(a, z) = \left[ \eta z^{1/\alpha} \lambda - r(\lambda - 1) \right] a
\]

The entry decision can be simplified to the comparison between the rate of return to wealth from entrepreneurship, \( \left[ \eta z^{1/\alpha} \lambda - r(\lambda - 1) \right] \) and the rate of return to savings \( r \).

Now suppose that the entrepreneur has to pay a tax rate on profit \( \tau \in (0, 1) \), so that total after tax profit is \( \pi = (1 - \tau) \left\{ \eta z^{1/\alpha} \lambda - r(k - a) \right\} > 0 \). The capital and labour demand functions remain unaltered, however the entry threshold becomes: \( z_{\text{min}}(\tau) = \left( \frac{z}{\eta} \left( 1 + \frac{r}{\lambda(1-\tau)} \right) \right)^{\alpha} \). Note that \( \frac{\partial z_{\text{min}}}{\partial \tau} > 0 \) so that taxes act as an entry deterrent to potential entrepreneurs. Also, compared to the threshold without taxation, again ignoring aggregates, taxation seems to induce less entrants at first pass. However, the wage effect makes this relationship ambiguous.\(^8\)

The effect of financial frictions here is twofold. The first effect works as in the case without taxes, through the wage equation (see below). However financial frictions exacerbate the taxation effect at the individual level, as is clear in the equation for \( z_{\text{min}} \) above. Taking the limit of \( z_{\text{min}} \) as \( \lambda \to \infty \) we see that this threshold is the same as in the case without taxation \( \left( z_{\text{min}} = \left( \frac{z}{\eta} \right)^{\alpha} \right) \). Given the nonlinearity of the expression above in both \( \lambda \) and \( \tau \), the combined effect of both taxation and financial frictions is especially pronounced at high levels of taxation and financial frictions, which is an empirical reality for much of the developing world. The intuition here is relatively simple. When financial frictions are low \( (\lambda \text{ is high}) \) the return to the project is high because individuals are not constrained in the amount of capital they can put into the project. In this sense \( \lambda \) is a determinant of the rate of return on capital for entrepreneurs. In this scenario taxes matter less, because they are reducing an already high rate of return. On the other hand, when financial frictions are high \( (\lambda \text{ is low}) \) the rate of return on the project is very sensitive to taxation.

\(^8\)I will forgo questions of government spending in this environment to focus on the issues at hand. One can imagine that government spending can be used to finance public goods that enter the utility function linearly in some capacity. As will be seen shortly corruption has significant negative effects on the amount of revenue the government can raise and the way it is raised. See Tanzi and Davoodi (1997).
2.1.1 Aggregation I

The aggregate labour demand equation is:

\[ L^d = \lambda \left( \frac{(1 - \alpha)}{w} \right)^{1/\alpha} E(z^{1/\alpha} | z > z_{\text{min}}(\tau)) E(a) (1 - F(z_{\text{min}}(\tau))) \]

which, together with a normalized labour supply \( L^s = 1 \) implies:

\[ w = (1 - \alpha) \left( \lambda E(a) \int_{z_{\text{min}}(\tau)}^{\bar{z}} z^{1/\alpha} f(z) \, \text{d}z \right)^{\alpha} \]

Note that \( \frac{\partial w}{\partial z_{\text{min}}} < 0 \); a higher entry threshold reduces labour demand at the extensive margin while leaving the intensive margin unaltered.

Each entrepreneur’s entry decision depends on the aggregate state through the wage rate \( w \). In order for aggregate outcomes to be consistent with individual decisions, it must be that each individual facing the threshold \( z_{\text{min}} \) results in an aggregate threshold that is consistent. More precisely, the existence of such a threshold requires that the equation

\[ z_{\text{min}} = \varphi(z_{\text{min}}) = \left[ \frac{\tau}{\alpha} \left( 1 + \frac{\tau}{\lambda(1 - \tau)} \right) \left( \lambda E(a) \int_{z_{\text{min}}}^{\bar{z}} z^{1/\alpha} f(z) \, \text{d}z \right)^{1 - \alpha} \right]^{\alpha} \]

has a fixed point.\(^9\) It is relatively straightforward to show that \( \varphi' < 0 \). Taking the limits of the function as \( z_{\text{min}} \) approaches the boundaries, we get

\[ \lim_{z_{\text{min}} \rightarrow z_{\text{bar}}} \varphi(z) = \left( \frac{\tau}{\alpha} \left( 1 + \frac{\tau}{\lambda(1 - \tau)} \right) \left[ \lambda E(a) E(z^{1/\alpha}) \right]^{1 - \alpha} \right)^{\alpha} > 0 \quad \text{and} \quad \lim_{z_{\text{min}} \rightarrow \bar{z}} \varphi(z) = 0 \]

which guarantees the existence of a fixed point.

Given the above setup, wealth levels evolve according to:

\[ a_{t+1} = \begin{cases} 
(1 - \gamma) \left[ (1 - \tau) \pi(a_t, z_t) + w_t + a_t \right] & \text{if } z_t \geq z_{\text{min}} \\
(1 - \gamma) \left[ w_t + (1 + r)a_t \right] & \text{else} 
\end{cases} \]

2.1.2 Corruption

Now consider a case in which, prior to making the entry decision and prior to drawing their productivity, entrepreneurs can negotiate with a bureaucrat who can lower their tax rate in exchange for a bribe.\(^10\) The bureaucrat is corruptible, and can be paid to assign a lower tax rate to the entrepreneur. There is a cost to the bureaucrat associated with being corrupt,

\[^9\] The equation is obtained by plugging in for \( \eta(w) \) in the equation for \( z_{\text{min}} \).

\[^{10}\] The intuition here is that entrepreneurs do not know ex-ante how good the project is but want to find out the tax rates they will be charged in order to know whether they should operate or not.
denoted by $\theta(a, \tau)$, where $a$ is the wealth level of the individual paying the bribe and $\tau$ the tax rate assigned to that individual. One can think of this cost as the expected loss to the bureaucrat in the (random) event that he is caught.\textsuperscript{11} It stands to reason that, all else equal, if higher wealth individuals bribe a bureaucrat, then the loss in revenue to the government is higher, which should lead to an increase in the probability of the bureaucrat getting caught and thus in the total cost to the bureaucrat.\textsuperscript{12} The same holds true for the tax rate that the bureaucrat charges, where the cost of corruption is decreasing in the tax rate charged. I assume that the bureaucrat is bounded below in the amount of tax he can charge by $\tau_l$, and above by the legally mandated, government announced rate of $\phi$ where $\tau_l < \phi$. The lower bound on the tax rate can be thought of as the lowest tax rate at which the bureaucrat is still able to be corrupt with the probability of being caught less than one. Put more succinctly; at any $\tau < \tau_l$, the bureaucrat is caught with probability one, and therefore is not willing to engage in corruption. If the bureaucrat charges the legally mandated rate of $\phi$, there is no corruption and therefore the costs are zero.\textsuperscript{13} The fall in costs due to increasing the tax rate $\tau$ is larger for wealthier types since the gain in revenues to the government is also larger. The above reasoning sets up the following assumption on the cost $\theta$.

**Assumption 1:** $\theta_a \geq 0$, $\theta_\tau \leq 0$, $\theta(\cdot, \phi) = 0$ and $\theta_{\tau a} < 0$.

The bureaucrat is unaware of the wealth level of each entrepreneur, but knows the distribution. Denote by $b(a)$ the bribe that the bureaucrat requests from type $a$ and $\tau(a)$ the tax rate that he assigns that type.\textsuperscript{14} Net revenue to the bureaucrat from each type is $R(a) \equiv b(a) - \theta(a, \tau)$. The bureaucrat then chooses a series of contracts, where each contract is a tuple, $< \tau(a), R(a) >$ to maximize expected revenue subject to:\textsuperscript{15}

\begin{align}
G(\tau(a)) [a - R(a) - \theta(a, \tau(a))] & \geq G(\phi)a \quad \forall a \\
G(\tau(a)) (a - R(a) - \theta(a, \tau(a))) & \geq G(\tau(a')) (a - R(a') - \theta(a', \tau(a'))) \quad \forall a, a' 
\end{align}

\textsuperscript{11}I will make this more explicit in the quantitative section.
\textsuperscript{12}Note that the cost to the bureaucrat to being caught need not be particularly onerous, it is sufficient that he lose all the proceeds from corruption if caught for the strategic component of what follows to be consistent.
\textsuperscript{13}If the bureaucrat charges $\phi$, he is beyond reproach because he followed the law and cannot be accused of malfeasance.
\textsuperscript{14}Types refer to wealth levels.
\textsuperscript{15}See the appendix for an explicit expression for $G(\cdot)$. Here I am suppressing the dependence of $G(\cdot)$ on other variables for ease of notation.
The individual rationality (IR) constraint is simply the participation constraint, where the ex-ante outside option for the entrepreneur is given by the expected utility of facing the tax rate $\phi$ with wealth holdings $a$. The incentive compatibility (IC) condition represents the local downward constraint. Note that the cost to the bureaucrat is based on the contract offered, not on the actual wealth level of the individual accepting the contract.

As a way of illustrating the problem’s outcome, consider a sequence of contracts $<\tau(a), R(a)>$. The shaded area in figure 2 depicts the set of $\tau$ and $R$ that are feasible for type $a^h$ given some $\tau(a')$ and $R(a')$ where $a^h > a'$. As the figure clearly shows, any contract offered to type $a^h$ must lie on or below the payoff curve that goes through $<\tau(a'), R(a')>$ which implies that the tax rate $\tau(a)$ is strictly decreasing in wealth. This sets up the following result:

**Proposition.** Consider a menu of contracts $<\tau(a), R(a)>$ that maximizes the bureaucrat’s revenue. Then the following hold:

a) $\tau(a)$ is weakly decreasing in $a$.

b) $U^*(a) = G(\phi)a$.

c) the optimal menu is: $<\bar{R}(a^*), \tau_l>, <0, \phi>$ where

\[
a^* = \arg\max_a (1 - H(a)) (\kappa(\tau_l)a - \theta(a, \tau_l))\]

and $\bar{R}(a) = \kappa(\tau_l)a - \theta(a, \tau_l)$.

d) an unique interior solution $a^*$ exists iff $\rho(a) < \frac{\kappa(\tau_l) - \theta(a, \tau_l)}{\kappa(\tau_l)a - \theta(a, \tau_l)}$

where $\rho(a) = \frac{h(a)}{1 - H(a)}$ is the hazard rate.

e) A mean preserving spread increases $a^*$ if $\rho$ is increasing in $a$.

Proof: See appendix.

### 2.1.3 A Numerical Example

To illustrate how inequality affects the menu of choices the bureaucrat offers through the threshold $a^*$ as well as the entry decision, consider the static case in which the wealth distribution is Pareto with parameter $\alpha$ and the cost function is $\theta(a, \tau_l) = (\phi - \tau_l)^2 (\mu a^2 + \mu)$. Suppose that productivity follows a normal distribution, in which case $a^*$ is given by:

\[
\frac{1}{v} = \frac{\kappa(\tau_l) - 2a^* (\phi - \tau_l)^2}{\kappa(\tau_l)a^* - (\phi - \tau_l)^2 (\mu a^2 + \mu)}
\]

\[16\)See appendix for a detailed explanation on the shape of the payoff curves.
I simulate a sequence of mean preserving wealth distributions and plot the dependence of a few key parameters on the dispersion of the wealth distribution, specifically the threshold $a^*$, the expected productivity threshold and the change in wealth at the top percentile of the distribution. Figure 3 depicts the dependence of $a^*$ on inequality. As the dispersion of the wealth distribution (inequality) increases, the bureaucrat has a very low quality ‘signal’ as to each individual’s location within that distribution. In order to insure himself against losing a large proportion of bribes, he sets the threshold relatively low. In this way inequality is persistent; in economies where wealth is equally distributed a very small proportion of the population faces favourable tax rates, while the larger mass of entrepreneurs pays the same marginal tax rate, thus maintaining a fairly equal distribution. To further elaborate on this point, figure 4 below plots the relationship between the percent change in the wealth holdings of the top 1 percent of the distribution and $1/u$, again as a proxy for inequality. Those at the top 1 percent of the distribution that do not operate have the same relative increase in income, so the differences depicted in figure 4 are purely due to the initial level of inequality. As inequality increases, the wealth threshold that the bureaucrat sets for accepting bribes $a^*$ falls, which then reduces the actual bribe each individual above this threshold pays to the bureaucrat $\kappa(\tau_l)a^*$. Therefore the top percentile of the distribution that actually operates not only gets to keep a larger percentage of their wealth/income because they pay lower taxes; they also have to pay less in bribes because

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$^{17}$Each point on the x-axis represents the inverse of the Pareto index $\nu$. 

Figure 3: A depiction of the wealth threshold as a function of inequality

the bureaucrat is insuring himself against setting too high a threshold.

Figure 4: % change for the top 1%

Figure 5 plots the dependence of expected productivity on inequality. The negative relationship between these variables reflects the fact that as inequality increases and the bureaucrat reduces the wealth threshold, the proportion of entrepreneurs that pay the low tax rate increases, thus reducing the barriers to entry for those that are allowed to bribe. In a highly equal society, bureaucrats have more precise information about an individual’s wealth holdings so that most entrepreneurs follow the rules and therefore the average entry threshold is higher with more egalitarian distributions.

It is worth reiterating at this point that entrepreneurs negotiate with the bureaucrat prior to the entry decision, and take aggregates as given. However, forward looking agents have all of the information required to infer aggregate outcomes, in this case the wage rate.
Note that a proportion $H(a^*)$ of the population will not pay bribes and will therefore face the legally mandated tax rate of $\phi$ if they decide to operate. These entrepreneurs enter only if $z \geq z_{\min}(\phi)$, and since $\phi > \tau_l$ their entry threshold is higher than those who bribe the bureaucrat ($z_{\min}(\tau_l)$). A similar argument to the one made in the case without corruption establishes the existence of a fixed point for both $z_{\min}(\tau_l)$ and $z_{\min}(\phi)$. Note that these are the two productivity thresholds potential entrepreneurs face in the presence of corruption. Given that $z_{\min}(\tau)$ is increasing in $\tau$, those entrepreneurs who are able to take advantage of the bribing opportunity will face lower (productivity) barriers to entry.\textsuperscript{18}

In this environment wealth evolves according to the following equations:

\begin{align}
    a_{t+1} &= (1 - \gamma)((1 + r)a_t + w_t) & \text{if } z_t < z_{\min}(\tau_l) \\
    a_{t+1} &= (1 - \gamma)((1 + r)a_t + w_t) & \text{if } z_{\min}(\tau_l) < z_t < z_{\min}(\phi) \& a_t < a_t^* \\
    a_{t+1} &= (1 - \gamma)(a_t + (1 - \phi)\tilde{\pi}(z, z_{\min}(\phi))a_t + w_t) & \text{if } z_t > z_{\min}(\phi) \& a_t < a_t^* \\
    a_{t+1} &= (1 - \gamma)((1 - \tau_l)\tilde{\pi}(z, z_{\min}(\tau_l))(a_t - \kappa a_t^*) + w_t) & \text{if } z_t > z_{\min}(\tau_l) \& a_t \geq a_t^*
\end{align}

where $\tilde{\pi}(z, z_{\min}(\tau)) = \lambda \eta E(z^{1/\alpha}|z > z_{\min}(\tau(a))) - r(\lambda - 1)$.

Since labour supply is inelastic, every individual earns labour income $w$. Those that are below the lowest possible threshold $z_{\min}(\tau_l)$ will not operate regardless of their standing in the wealth distribution so they earn labour and rental income. This is described by the first equation above. Those individuals above $z_{\min}(\tau_l)$ but below $z_{\min}(\phi)$ will operate if and only if they hold enough wealth to bribe the bureaucrat ($a \geq a^*$), otherwise they face the

\textsuperscript{18}See the appendix for aggregate equations.

Figure 5: Expected Productivity and Inequality
legally mandated tax rate $\phi$ and thus don’t find it worthwhile to become entrepreneurs. The individuals below the wealth threshold are represented by the second equation. The third equation describes those that are not able to pay the bribe because they fall below the wealth threshold, but operate nonetheless because their productivity is sufficiently high. The fourth equation describes those that are wealthy enough to pay the bribe and therefore will always face the lowest threshold $z_{\text{min}}(\tau_t)$.

Distortions coming from corruption here are twofold: first when comparing two economies that differ only in their levels of corruption, the more corrupt economy has higher entry at the lower end of the productivity distribution (individuals described by line four above). Second if corruption is sufficiently high and the marginal product of labour is heavily dependent on capital, a corrupt economy will have inflated wages, which increases the higher productivity threshold ($z_{\text{min}}(\phi)$). This results in those potential entrepreneurs that fall below the wealth threshold $a^*$ to face higher entry barriers where corruption is more severe. This second effect reduces the number of mid productivity types entering entrepreneurship, and if inequality is significant, reduces the mass of entrepreneurs in the middle of the distribution.

3 Calibration and Counterfactual Experiments

I consider the effects of corruption on aggregates by calibrating the model to a benchmark economy with no corruption. For the benchmark economy I choose Sweden. There are three main adjustments to the model described above; first, I assume that the productivity parameter $z$ is no longer i.i.d but evolves according to an AR(1) process with persistence parameter $\rho$ and log normal error term.$^{19}$ Second, I assume that individuals are heterogeneous in labour income, and that their labour productivity is perfectly correlated with their general productivity (knowledge) parameter $z$. In the calibrated model the source of heterogeneity is still $z$, but its effect is now spread across two dimensions; the ability to run a project, which was the initial interpretation of $z$ as well as labour productivity. Wages are then quoted in units of effective labour instead of just units of time. Third, I endogenize the lower bound of the tax rate that the bureaucrat charges when he accepts

---

$^{19}$This structure has implications for the informational structure of the screening problem outlined above. However, to keep things simple I will assume that there is no retention of information and each generation starts anew, with bureaucrats being born each generation without any knowledge of previous history.
bribes, denoted by the parameter \( \tau_i \). The revenue maximization problem for the bureaucrat in this scenario is two dimensional, and corruption levels affect not only the wealth threshold \( a^* \) but also the taxation levels that result from bribery.

After having calibrated the benchmark economy, I perform a series of counter factual experiments. First I increase the level of corruption in the benchmark economy and observe the responses of inequality as measured by the GINI coefficient as well as the percentage of income earned by the top of the distribution. I also report the effect of corruption on the lower bound of the tax rate \( \tau_i \) as well as the size distribution of firms according to both effective labour and productivity. I then increase the dispersion of the productivity distribution and observe the effect on the size distribution of firms. As a check on the quantitative significance of financial frictions, I increase financial frictions by reducing the parameter \( \lambda \) and document the difference in the responses of the above variables. Finally, I decompose the corruption inequality feedback loop, to quantify the effect inequality has on corruption.

### 3.1 Calibration

The value of the calibrated parameters and their sources are given in table 1 below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>persistence of productivity</td>
<td>0.677</td>
<td>De Nardi 2003, Björklund/Jäntti 1997</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>within generation prod. variance</td>
<td>0.13</td>
<td>% of entr (GEM) 8.9%</td>
</tr>
<tr>
<td>( \phi )</td>
<td>upper bound on tax rate</td>
<td>37.5%</td>
<td>WB corporate rate on distributed profits</td>
</tr>
<tr>
<td>1 - ( \gamma )</td>
<td>fraction of wealth bequeathed</td>
<td>&gt; 0.51</td>
<td>De Nardi 2003, Laitner Ohlsson 1997</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>coefficient of financial frictions</td>
<td>1.75</td>
<td>Firm leverage, Song 2005</td>
</tr>
</tbody>
</table>

The persistence of the productivity parameter (\( \rho \)) is assumed to be the same in Sweden as in the US as in De Nardi (2003). The source of this assumption is Björklund and Jäntti.
(1997), who do not reject the hypothesis that income mobility in Sweden is the same as in the US. The original source of the value is Zimmerman (1992). There is a large literature that attempts to estimate different variants of the parameter $\rho$. Both Solon (1992) and Zimmerman (1992) for example estimate that the income correlation across generations is at least 0.4, but probably higher. A persistence of 0.677 implies a wage income correlation of 0.6 across generations in this model, which seems to be relatively close to intergenerational estimates of income correlation.

I use the percentage of the population that are entrepreneurs obtained from the Global Entrepreneurship Monitor’s database to target the variance of the productivity distribution $\sigma^2$. The value reported above for Sweden is very close to the value of 0.14 reported in De Nardi (2003). The value of $\phi$ is sourced from the World Bank and represents the profit redistribution tax rate. The parameter $\gamma$ is chosen to target the ratio of inheritance to wealth in Sweden from Laitner and Ohlsson (1997).\textsuperscript{20} In the model the only reason for inheritance is the bequest motive, and accidental bequests are not considered. However the motivation for bequests is irrelevant to the outcome of the model; inequality persistence is present whether bequests are accidental or otherwise.

Given that the model assumes a small open economy, an assumption that describes the chosen benchmark economy well, the interest rate is not endogenously determined. To target the financial frictions parameter $\lambda$, I use firm leverage as a percentage of total assets reported in the dataset of 6000 Swedish firms reported by Song (2005). The dataset covers a variety of firms with sales above 10,000 Swedish kroner and therefore firm sizes are well represented. The average leverage ratio for Swedish firms is around 75\%, which implies a $\lambda$ of 1.75 for those firms who operate.

I assume the functional form for the corruption cost function to be $\theta(a, \tau) = (\phi - \tau)^2(v_1a + v_2)$ where $v_2$ is simply a normalization parameter. To estimate this function I suppose that the costs to the bureaucrat are the expected losses from being caught. I then normalize the World Bank Governance Institute’s control of corruption index to use it as a proxy for the probability that a bureaucrat is caught being corrupt. Denote by $\omega$ the probability of being caught, then we have

$$b(a) - \theta(a, \tau_t) = (1 - \omega)b(a)$$

\textsuperscript{20}The ratio of 51\% seems to be a lower bound on the parameter because of issues with data quality.
where the right hand side is the expected bribe that the bureaucrat retains. I then use the identity \( \frac{\theta(a, \tau_l)}{\theta(a)} = \omega \) where \( b(a) = \kappa(\tau_l) a^* \) to target the parameters of the cost function.

### 3.2 Responses to Increased Corruption

*Figures 6(a-c)* below depict the effect of corruption on the changes in the GINI coefficient on wealth, \( \tau_l \) and the income that goes to the top 1 percent of the distribution respectively.

For the graphs relating to the GINI coefficient and the share of income that goes to the top 1% of the distribution, the *y*-axis depicts the differences in these measures between the corrupt economy and the benchmark one, where the level of corruption is measured in the *x*-axis. *Figure 6 c* shows that corruption can account for up to 7% of aggregate income being reallocated to the top 1 percent of the income distribution. The non-linear relationship depicted in the first figure is also an interesting feature of the outcome, inequality is more sensitive at higher corruption levels, even if the cost function \( \theta \) is linear in wealth. The direct effect here comes from the lower bound on taxes \( \tau_l \), as corruption increases and costs to being corrupt (probability of being caught) fall, bureaucrats become more brazen.
in their promises to lower rates and because the function is quadratic in \( \Delta \equiv \phi - \tau_l \), at high corruption levels the lower bound on the tax rate falls significantly. This is made clear in figure 6 b, and the response of \( \tau_l \) as corruption increases confirms the assertion above. As can be seen from the figure 6 a, the GINI coefficient follows the behaviour of the upper tail of the distribution.

The main source of inequality in this setting is at the entrepreneurial level, and over time some entrepreneurs end up accumulating a significant amount of wealth so that inequality is being generated by the upper tail of the distribution. The differences between a corrupt and clean economy depicted in figures 6 a-c above are a result of the low taxes the upper tail of the distribution is paying, as well as the fact that low wealth individuals are less likely to operate due to the barriers they face. If we observe how the thresholds \( z_{\min}(\phi) \) and \( z_{\min}(\tau_l) \) evolve over time we get figure 7 below. The dotted red line represents \( z_{\min}(\phi) \), the thresholds that those below \( a^* \) wealth face in becoming entrepreneurs. The solid blue line is the threshold faced by all in an economy with \( \omega = 1 \) (probability of being caught). The solid red line represents \( z_{\min}(\tau_l) \), the threshold faced by those who end up paying the bribe. From the figure one can see the patterns of entry; those individuals above the solid blue line but below the dotted red line are the counterfactual mass of entrepreneurs that is missing in the corrupt economies, and, if some of them are unable to pay bribes because of their position in the wealth distribution, they will not enter, thus reducing the mass of firms in the middle of the productivity distribution. At the upper tail of the productivity distribution, (above the dotted red line) there is no difference between the two economies as all enter, but at the lower end, it is clear that the corrupt economy will have a thicker lower tail given that some low productivity, high wealth types will operate projects that would not have been operated in the absence of corruption. Given the above discussion on entry, it is interesting to note the differences in the size of the entrant firms as measured by labour.\(^{21}\) This is motivated by Tybout (1998). In highly corrupt countries, the tails at both the top and the bottom of the distribution are thick at the expense of the middle. How does this model fit this empirical fact? Table 3 below is a representation of such differences. While the absolute values for each entry are not essential because they were not specific targets, it is important to note that the entry discussion above is quantitatively important when we consider relative differences. In a corrupt economy the lower end of

\(^{21}\) Labour is measured in units of effective time.
As the numerical exercise at the beginning of the paper made clear, the effects of corruption are more significant when inequality is particularly severe. The productivity distribution for Sweden is highly concentrated around the mean and so the table above underestimates the effect of corruption on some of the economies where inequality in opportunity (productivity) is more pronounced. To quantify this effect consider the table below, where I have increased the variance of the productivity distribution to 0.3:

**TABLE 4:** Labour size distribution ($\sigma^2 = 0.3$)

<table>
<thead>
<tr>
<th>labour</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = 1$</td>
<td>31%</td>
<td>32%</td>
<td>37%</td>
</tr>
<tr>
<td>$\omega = 0.4$</td>
<td>63%</td>
<td>13%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Whereas in the first instance medium size enterprises in the corrupt economy were at 67% of the benchmark uncorrupt one, when we increase $\sigma$ we see that ratio shrink to nearly 22

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Note that $\omega = 0.4$ maps into a value of 0.6 on the normalized World Bank Governance Index, which is the corruption index of a large proportion of the world’s population.
40%, a significant fall given that \( \sigma \) is probably still relatively small. Also, the lower tail doubles in size with higher variance.

Financial frictions are essential for the distortionary effect of corruption on the size distribution of firms and aggregate productivity. Without financial frictions, taxation is immaterial in the individual entry decision, and, given that it works through taxation, so is corruption. In this sense, financial frictions attenuate the effect of corruption on inequality and the effect on productivity through the entry decision. In order to quantify these effects I lower the coefficient of financial frictions \( \lambda \) and compare the response of the GINI coefficient, the lower bound on the tax rate and the share of income allocated to the top 1% of the income distribution in the two environments (figure 6 d-f). The effect of increasing financial frictions is significant, especially for the lower bound on taxes. In this way financial frictions do not only have first order effects of their own, but they tend to exacerbate the effects of other frictions such as inequality and corruption. Without financial frictions there is very little linking income and wealth, but when the two are linked because of collateral requirements, the effect of other frictions kicks in and can account for a large fraction of income and wealth inequality as well as the disparity in the size distribution of firms.

3.3 Decomposing the Effects of Corruption on Inequality

The discussion in the numerical example in section 2.1.4 made the case that the screening mechanism produces feedback effects between corruption and inequality. More specifically, increases in inequality reduce the wealth threshold above which the bureaucrat is willing to accept bribes, thus increasing the frequency of corrupt transactions which further increases inequality. However, the counter factual experiments above do not distinguish between the direct effect of corruption on inequality and this indirect feedback effect. Since this mechanism is important in the process of inequality persistence, it is worthwhile to try to measure it in order to understand its quantitative properties.

To achieve this decomposition consider the following thought experiment. Suppose the steady state equilibrium has been computed and we calculate the equilibrium bribe, wealth threshold, \( \tau_i \) and \( \kappa(\tau_i) \). From the wealth threshold \( a^* \) it is easy to compute the equilibrium fraction of individuals that actually pay the bribe. The screening outcome predicts that this fraction is dependent on inequality itself and is the channel through
which the feedback mechanism amplifies the effects of corruption. Now suppose that we fix this fraction and, instead of letting individuals self-select whether to bribe depending on their wealth levels, we assign each individual a probability that he or she will pay the bribe, where this probability is the fraction of individuals that pay the bribe in the steady state. The amount each individual bribes is fixed to the value calculated in the steady state. In this experiment the wealth threshold is irrelevant because, given it is optimal to bribe, an individual’s position in the wealth distribution does not determine whether or not they pay the bribe. Therefore, the only effect of corruption on inequality is direct in this experiment. Figure 8 below depicts the effect of corruption on inequality as measured by both the differences in the GINI coefficient between the corrupt and the ‘clean’ benchmark economy and the differences in the amount of income that goes to the top 1% of the wealth distribution.

![Graphs showing the effects of corruption on inequality](image)

Figure 8: Decomposing the effects of corruption on inequality.

The solid lines in figure 8(a,b) represent the total effect of corruption on inequality as in figure 6. The dashed lines represent the outcome of the experiment outlined above. figure 8(c) plots the fraction of the difference in inequality that is explained by the feedback effect. At low levels of corruption, the direct effect of corruption on inequality accounts for up to 60 percent of the total, with the difference explained by the feedback effect of
inequality on corruption. At high levels of corruption however, the picture changes, with roughly 80 percent of the total effect explained by the feedback channel. This suggests that the modeling choice here is highly relevant, and that the quantitative effects would be incomplete in any model which ignores these feedback effects. Given that a lot of the inequality differences are generated by differences in wealth at the top 1% of the distribution, high levels of corruption generate larger differences in the wealth held by this group between the corrupt and benchmark economies. Higher levels of inequality then affect the frequency of corrupt transactions through the wealth threshold and, as explained in section 1.4, increase the persistence of inequality. In short, because higher levels of corruption produce more inequality, the persistence (feedback) effect is higher where corruption is higher.

4 Discussion

I have constructed a model of corruption and inequality with financial frictions in which inequality plays a significant role in the level of corruption and vice versa. Societies with higher concentrations of wealth end up being more corrupt because inequality induces bureaucrats to charge lower bribes to the higher end of the wealth distribution, but also to allow a larger proportion of the top of the wealth distribution to face lower taxes. This can explain to some extent the observed link between inequality and corruption, and it can do so without appeal to redistribution and government programs. In this environment, when wealth is not publicly observable and bureaucrats are corrupt, it is clear that any form of redistribution is going to be difficult to implement, even the targeted kind. The model seems to fit well the empirical facts regarding the size distribution of firms. More specifically, it goes a long way in explaining the puzzle of the ‘missing middle’ in the size distribution of firms.

This model has significant implications for the provision of the public good. When corruption and inequality are severe, government revenue suffers. Any redistributive scheme that promises government services or simple wealth transfers based on projections of what governments should collect rather than what they do collect is going to come up short. In order to remedy this problem some governments turn to borrowing in the international markets, but most simply reduce the amount of public goods provided. Since govern-
ment services are often targeted to alleviating poverty and providing better opportunities through education, this hurts the poor disproportionately. Through this channel the model has something to say about the persistence of poverty as well.

The model has no explicit growth components to it, but its implications for growth would be significant. For example the paper assumes that whether a generation operates or not is irrelevant for the productivity of the next generation. This assumption could be relaxed to suppose that operating has tangible benefits besides the effects of increasing one’s income and wealth. The children of successful parents are more likely to be successful themselves. If we equate success with operating a profit making enterprise, then it is possible that by putting larger hurdles for low wealth individuals to overcome, corruption is lowering growth rates, both for capital and aggregate output. Future work in this direction could reveal some salient features of corruption and its links to growth.
5 Appendix

5.1 A note on payoff curves

As noted above, tax rates not only affect the net profit of the entrepreneur, they also affect the entry decision. Denote by \( q(\tau(a)) = P[z > z_{\text{min}}(\tau(a))] \) the probability that an individual operates given some tax rate \( \tau(a) \). The utility of an individual that pays the bribe is:

\[
U = \begin{cases} 
\delta \{(1 - \tau(a))\bar{\pi}(z, z_{\text{min}}(\tau))(a - b(a)) + w + (a - b(a))\} & w/p \quad q(\tau(a)) \\
\delta \{(1 + \tau)(a - b(a)) + w\} & w/p \quad 1 - q(\tau(a)) 
\end{cases}
\]

If the individual decides not to participate, the utility is:

\[
U = \begin{cases} 
\delta \{(1 - \phi)\bar{\pi}(z, z_{\text{min}}(\phi))a + w + a\} & w/p \quad q(\phi) \\
\delta \{(1 + \tau)a + w\} & w/p \quad 1 - q(\phi) 
\end{cases}
\]

where \( \bar{\pi}(z, z_{\text{min}}(\tau)) = \lambda \eta E(z^{1/\alpha}|z > z_{\text{min}}(\tau(a)) ) - r(\lambda - 1) \) and \( \tau_l \leq \tau(a) \leq \phi \). Denote by \( G(\tau) \) the function:

\[
G(\tau) = q(\tau) \{(1 - \tau) \left[ \lambda \eta E(z^{1/\alpha}|z > z(\tau)) - r(\lambda - 1) \right] + 1 \} + (1 - q(\tau))(1 + r)
\]

Using the methodology of Maskin and Riley (1984), note that expected utility is given by \( U = G(\tau(a)) [a - R(a) - \theta(a, \tau(a))] \). Total differentiation gives \( dU = \frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial R} \frac{\partial R}{\partial \tau} = 0 \) to get \( \frac{\partial R}{\partial \tau} |_{\tau=0} = - \frac{\partial U/\partial \tau}{\partial U/\partial R} = (a - R(a) - \theta(a, \tau)) G' - \theta G \). Given that \( \theta(\cdot, \phi) = 0 \) we have \( \frac{\partial R}{\partial \tau} |_{\tau=\phi} < 0 \). By monotonicity of \( G \) and \( \theta \) we know that \( \frac{\partial R}{\partial \tau} \) is not always positive for all \( \tau \).

Figure 9 is an illustration of a possible set of payoff curves for a given type \( a \). It could be that \( \frac{\partial R}{\partial \tau} \) is positive for all of the curves, or for none of them, in this case I have illustrated some that have a positive slope for some interval. Consider a case for a type \( a \). Take two different utility levels \( U_1 \) and \( U_2 \), where \( U_2 > U_1 \). At each tax level, \( R_1 > R_2 \), which implies that the curve for \( U_2 \) lies entirely under that for \( U_1 \) for type \( a \). Furthermore since \( R_1 > R_2 \) then we have that \( \frac{\partial R}{\partial \tau} |_{\tau=\pi_2} < \frac{\partial R}{\partial \tau} |_{\tau=\pi_1} \) for all \( \tau \), which gives the curves the shape in the figure below.

Denote by \( \tilde{R}(a) \) the revenue level extracted from type \( a \) such that it gives this type the outside option at \( \tau = \tau_l \) so that \( G(\tau_l) \left[ a - \tilde{R}(a) - \theta(a, \tau_l) \right] = G(\phi)a \). Rearranging we get

\[
\tilde{R}(a) = a \left( 1 - \frac{G(\phi)}{G(\tau_l)} \right) - \theta(a, \tau_l)
\]

\[23\] Note that the bribe here is paid ex-ante.
Figure 9: Payoff curves for an arbitrary type $a$. $U_1 < U_2 < U_3$.

Given the above discussion, if $\frac{\partial \tilde{R}}{\partial \tau} \big|_{\tau = \tau_i} < 0$ for some type $a$, we have that $\frac{\partial R}{\partial \tau} < 0$ for all utility levels for that type, i.e. the payoff curves are strictly downward sloping. Note that $\frac{\partial \tilde{R}}{\partial \tau} \big|_{\tau = \tau_i} = \left(a - \tilde{R}(a) - \theta(a, \tau_i)\right) G'(\tau_i) = \frac{G(\phi)}{G(\tau_i)} a G'(\tau_i) < 0$ where the second equality comes from the definition of $\tilde{R}$ and the inequality from lemma 5. Since this result is not dependent on $a$, it implies that payoff curves are strictly downward sloping for all types.

Consider a case with two types, $a^h$ and $a^l$ where $a^h > a^l$. The combinations of $R$ and $\tau$ that give the high type the outside option (the outside option curve) lie strictly above those that give the low type the outside option if and only if $\kappa(\tau) > \theta_a(a, \tau)$ for all $\tau \in [\tau_l \phi]$ where $\kappa(\tau) = 1 - \frac{G(\phi)}{G(\tau)}$. To see why, note that the curve for type $i$ is described by $R^i = \kappa(\tau)a_i - \theta(a_i, \tau)$. We need $R^h - R^l > 0$, which implies that $\kappa(\tau) > \frac{\theta(a^h, \tau) - \theta(a^l, \tau)}{a^h - a^l}$. Taking limits we get the requirement $\kappa(\tau) > \theta_a(a, \tau)$ which sets up the following assumption:

**Assumption 2:** $\kappa(\tau) > \theta_a(a, \tau)$ for all $\tau$ and $a$. 

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Figure 10 depicts the outside option curves for the lowest and highest type.

![Figure 10: Outside option curves for $a^b$ and $a^l$.](image)

### 5.2 Proofs

**Proof.** The dependence of the function $G$ on $\tau$ is essential for the results that follow, therefore it is useful to establish the following result $\frac{\partial G}{\partial \tau} < 0$.

\[ G(\tau) = (1 - F(z_{\min}(\tau))) [\eta(1 - \tau) \{E[z \mid z > z_{\min}(\tau(a))] - r\} - r\tau] + (1 + r) \]
\[ = (1 - F(z_{\min}(\tau))) [\eta(1 - \tau) E[z \mid z > z_{\min}(\tau(a))] - r] - r (1 - F(z_{\min}(\tau))) \tau + (1 + r) \]

The derivative of the second part is obviously negative, so we just need to show that the first part is monotonely decreasing in $\tau$.

First we find the conditional distribution of $z$. $Pr[z \mid z > z_{\min}(\tau)] = \frac{Pr[z_{\min}(\tau) < z < x]}{(1 - F(z_{\min}(\tau)))}$ and the pdf is: $\frac{f(z)}{(1 - F(z_{\min}(\tau)))}$. So that $E[z' \mid z > z_{\min}(\tau(a))] = \frac{F(x) - F(z_{\min}(\tau))}{(1 - F(z_{\min}(\tau)))} \int_{z_{\min}(\tau)}^{z} zf(z) dz$. Rearranging the terms we get:

\[ G(\tau) = \eta (1 - \tau) \int_{z_{\min}(\tau)}^{z} zf(z) dz - rF(z_{\min}(\tau)) \]
\[ \frac{\partial}{\partial \tau} \int_{z_{\min}(\tau)}^{z} zf(z) dz = -z_{\min}(\tau)f(z_{\min}(\tau)) \frac{\partial z_{\min}(\tau)}{\partial \tau} \text{ by the Leibniz rule.} \]

Taking derivatives we have

\[ \frac{\partial G(z_{\min}(\tau))}{\partial \tau} = -\eta \int_{z_{\min}(\tau)}^{z} zf(z) dz - \eta (1 - \tau) z_{\min}(\tau)f(z_{\min}(\tau)) \frac{\partial z_{\min}(\tau)}{\partial \tau} + rf(z_{\min}(\tau)) \frac{\partial z_{\min}(\tau)}{\partial \tau} = \]
\[= -\eta \int_{z_{\text{min}}}^{z} zf(z)\partial z + f(z_{\text{min}}(\tau)) \frac{\partial z_{\text{min}}(\tau)}{\partial \tau} [r - \eta(1 - \tau)z_{\text{min}}(\tau)] = -\eta \int_{z_{\text{min}}}^{z} zf(z)\partial z < 0\]

a) The proof is similar to Maskin and Riley (1984).

b) Consider a sequence of wealth types where: \(\underline{a} < a_2 < \ldots < \bar{a}\). Here we show that
\(< \tau(a_i), R(a_i) > \succ_{a_i} < \tau(a_{i-1}), R(a_{i-1}) > \). Suppose that this is not true, then we have that
\(< \tau(a_i), R(a_i) > \succ_{a_i} < \tau(a_{i-1}), R(a_{i-1}) > \). Since \(< \tau(a_{i-1}), R(a_{i-1}) > \) is optimal for type 
\(i - 1\). This then implies \(< \tau(a_{i-1}), R(a_{i-1}) > \succ_{a_{i-1}} < \tau(a_k), R(a_k) > \) for all \(k \leq i - 1\) by
the assumption above and the first part of the proof. This then implies;
\(< \tau(a_{i-1}), R(a_{i-1}) > \succ_{a_i} < \tau(a_k), R(a_k) > \) for all \(k \leq i - 1\). Now consider a scheme
that keeps the same \(\tau(a_i)\) for all \(i\) but increases \(R\) to \(\bar{R} = R + \delta\) for all \(k \geq i\). For \(\delta\)
small enough, \(< \tau(a_{i-1}), R(a_{i-1}) > \succ_{a_i} < \tau(a_k), \bar{R}(a_k) > \). Therefore there exists another
contract that gives the bureaucrat higher wealth. This is a contradiction. Therefore:
\(< \tau(a_i), R(a_i) > \succ_{a_i} < \tau(a_{i-1}), R(a_{i-1}) > \). By this line of argument, the lowest level \(a\)
gets no surplus so that \(< \tau(a), R(a) > \succ_{a} < \phi, 0 > \) which gives us \(b\).

c) We need to show that the optimal menu for the bureaucrat to offer is: \(< \bar{R}(a^*),
\tau_1 >, < 0, \phi > \) where \(a^* = \arg \max_a (1 - H(a)) (\kappa a - \theta(a, \tau_1))\) and \(\bar{R}(a) = \kappa a - \theta(a, \tau_1)\) is
the revenue that gives type \(a\) the outside option at \(\tau = \tau_1\), and \(\kappa(\tau_1) \equiv 1 - \frac{G(\phi)}{G(\tau_1)}\). We do
this in three steps:

Step 1: Show that \(\tau \in \{\tau_1, \phi\} \) in any optimal contract. Suppose that there exists
an optimal menu that offers a contract \(< \bar{R}(a), \tau(a) > \) where \(\tau \in (\tau_l, \phi)\) for some types
\(a \in [\underline{a} \bar{a}]\). Consider first the contract offered to type \(\bar{a}\). Given the fact that this is the
highest type and increasing revenue does not affect the \(IC\) and \(IR\) constraints of types
lower than \(\bar{a}\), \(\tau(\bar{a}) = \tau_1\). Now consider the contracts offered to the next two lower types,
\(a_k\) and \(a_{k-1}\). Suppose that \(\tau(a_k), \tau(a_{k-1}) \in (\tau_1, \phi)\). Note that given the result in part a,
\(\tau(a_k) \in [\tau_l, \tau(a_{k-1})]\). Consider a small \(\varepsilon\) increase in \(\tau(a_k)\) such that \(\varepsilon \in \varepsilon\) where \(\varepsilon\) is such
that \(\frac{\partial \bar{R}}{\partial \tau}\bigg|_{\varepsilon=0}\) is constant for all \(\varepsilon \in \varepsilon\). Since the \(IR\) and \(IC\) constraints for all \(a < a_k\) are
unaffected by this small change in \(\tau(a_k)\), the only revenue that is affected is that of type \(a_k\)
and \(\bar{a}\). Denote by \(\Delta_{a_i}\) the change in revenue for type \(a_i\). Given that \(\frac{\partial \bar{R}}{\partial \tau}\bigg|_{\varepsilon=0} < 0\) we have
that \(\Delta_{\bar{a}} > 0\) and \(\Delta_{a_k} < 0\). Now since \(< \bar{R}(a), \tau(a) > \) is optimal it must be that this increase
in \(\tau(a_k)\) results in an aggregate loss in revenue so that \(|\Delta_{\bar{a}}| < |\Delta_{a_k}|\). If that is the case,
given that \(\Delta_{\bar{a}}\) is constant and that the indifference curves are strictly downwards sloping,
\[ \exists \varepsilon \in \epsilon \text{ such that } |\Delta a| > |\Delta a_k| \text{ if we reduce } \tau(a_k) \text{ by } \varepsilon. \] Therefore \( \tau(a_k) \in \{\tau_l, \tau(a_{k-1})\} \).

If that is the case, one can make the same argument to show that \( \tau(a_{k-1}) \in \{\tau_l, \tau(a_{k-2})\} \) and so on until the last type that is included in the contract \( \hat{a} \). Since \( \hat{a} \) will receive his outside option, it is easy then to see that \( \tau(\hat{a}) \in \{\tau_l, \phi\} \) which implies the desired result.

From now on, when we refer to a contract, it is implied that \( \tau = \tau_l \) whenever \( R \neq 0 \).

**Step 2:** Show that for any optimal contract \( R(a) \in [\hat{R}(a) \, \hat{R}(\hat{a})] \) \( \forall a \in [a \, \bar{a}] \) where \( \hat{R}(a) \) denotes the revenue that gives type \( a \) the outside option at \( \tau = \tau_l \). \( \hat{R}(a) = \kappa a - \theta(a, \tau_l) \).

Suppose that \( R(a) < \hat{R}(a) \) for some \( a > a_l \), then setting \( R(a) = \hat{R}(a) \) would still violate neither IC or IR and increase revenue so that \( R(a) < \hat{R}(a) \) is not optimal.

On the other side of the segment, suppose \( R(a) > \hat{R}(\hat{a}) \) for some \( a < a < \bar{a} \). then \( R(a) = 0 \) given that it violates the IR condition for type \( a \). By setting \( R(a) = \hat{R}(\hat{a}) \) the bureaucrat can increase revenue so that \( R(a) > \hat{R}(\hat{a}) \) is never optimal.

**Step 3:** Here we finally prove the main result.

Suppose that there exists another menu \( < R(a), \tau_l > \) where \( a \in A \subseteq [a \, \bar{a}] \) that maximizes revenue for the bureaucrat, where \( R(a) \in [\hat{R}(a_{min}) \, \hat{R}(a_{max})] \) as per step 2 where \( a_{min} = \min A \) and \( a_{max} = \max A \). Also \( R(a) \neq R(a^*) \) \( \forall a \in A \) where \( a^* \) is as defined in the proposition. Denote by \( R^a = \min(R(a)) \) for all \( a \in A \). Then by the monotonicity of \( \theta \) there exists an \( \hat{a} \) such that \( R^a = \hat{R}(\hat{a}) \). Take any \( \hat{a} \geq a \). The payoff to this type is given by

\[ G(\tau_l) (\hat{a} - R(\hat{a}) - \theta(\hat{a})) \],

while the payoff given by \( \hat{R}(\hat{a}) \) is \( G(\tau_l) (\hat{a} - \hat{R}(\hat{a}) - \theta(\hat{a})) \).

Since \( \hat{a} \leq \hat{a} \) then \( \theta(\hat{a}) \leq \theta(\hat{a}) \) and \( R(\hat{a}) \geq \hat{R}(\hat{a}) \), which implies that \( R(\hat{a}) \) violates the IC condition for type \( \hat{a} \). This in turn implies that the optimal contract choice for type \( \hat{a} \) is \( \hat{R}(\hat{a}) \).

Since the choice of \( \hat{a} \) was arbitrary, this is true for all \( a \geq \hat{a} \). In that case the revenue to the bureaucrat is:

\[ (1 - H(\hat{a}|a_{min} \leq a \leq a_{max})) \kappa(\hat{a} - \theta(\hat{a}, \tau_l)) \text{ where } a \in A. \]

However, since we assumed that \( R(a) \neq R(a^*) \) then it is clear that the bureaucrat can increase revenue by offering \( R(a^*) \) to all \( a \geq a^* \) which proves the result.

**d)** Note that the FOC is:

\[ -h(a) (\kappa(\tau_l)a - \theta(a, \tau_l)) + (1 - H(a)) \kappa(\tau_l) - \theta(a, \tau_l)) = 0. \]

For an interior solution to exist, given the monotonicity of \( \theta \), the FOC must be positive at the lower bound \( a_l \), which gives us the desired result. If this condition fails to hold then the solution is \( a = a_l \).

**e)** Suppose \( \rho \) is increasing in \( a \). Consider a mean preserving spread with cdf \( \bar{H}(a) = H(a + \xi) \) where \( \xi \) has mean zero. Denote by \( \hat{a} = \arg \max_a \left(1 - \bar{H}(a)\right) \left(\kappa a - \theta(a, \tau_l)\right). \)

Suppose \( \hat{a} < \bar{a} \), then it must be that \( f'(\bar{a}) = f'(\hat{a}) \) given that \( f'' = 0 \). Also, since \( f' > 0 \)
then $f(\tilde{a}) < f(\hat{a})$. By the FOC, we have that $0 = f'(\tilde{a}) - \rho(\tilde{a}) f(\tilde{a}) = f'(\hat{a}) - \rho(\hat{a}) f(\hat{a}) \Rightarrow \rho(\hat{a}) f(\hat{a}) = \rho(\tilde{a}) f(\tilde{a})$ which implies that $\rho(\hat{a}) > \rho(\tilde{a})$ which is a contradiction. \hfill \Box

5.2.1 Aggregation II

Aggregate demand is given by:

$$L^d = \lambda \left( \frac{(1 - \alpha)}{w} \right)^{1/\alpha} \left\{ [1 - F(z_{\min}(\tau_l))] E[z^{1/\alpha} | z > z_{\min}(\tau_l)] E[a - \kappa(\tau_l) a^* | a > a^*] [1 - H(a^*)] + \right.$$  
+ $E[z^{1/\alpha} | z > z_{\min}(\phi)] E[a | a < a^*] H(a^*) [1 - F(z_{\min}(\phi))]$ 

where the first part of the expression is the aggregate labour demand of those who are above the threshold $a^*$ and are therefore able to lower their tax rates. After some transformation:

$$L^d = \lambda \left( \frac{(1 - \alpha)}{w} \right)^{1/\alpha} \left[ I(z^{1/\alpha}, z(\tau_l), \tilde{z}) I(a - \kappa(\tau) a^*, a^*, \tilde{a}) + I(z^{1/\alpha}, z(\phi), \tilde{z}) I(\tilde{a}, a^*) \right]$$

where $I(y_1, y_2, y_3) = \int_{y_2}^{y_3} y_1 g(x) \partial x$ and $g(\cdot)$ is the appropriate pdf. The labour demand equation implies the following equation for wages and thresholds in the aggregate:

$$w = (1 - \alpha) \left[ \lambda I(z^{1/\alpha}, z(\tau_l), \tilde{z}) I(a - \kappa(\tau) a^*, a^*, \tilde{a}) + \lambda I(z^{1/\alpha}, z(\phi), \tilde{z}) I(\tilde{a}, a^*) \right]^{\alpha}$$

$$z_{\min}(\tau_l) = \frac{r \tau_l}{\lambda (1 - \tau_l)} \left( \frac{r}{\alpha} \left[ \lambda I(z^{1/\alpha}, z_{\min}(\tau_l), \tilde{z}) I(a - \kappa(\tau) a^*, a^*, \tilde{a}) + \right. \right. \right.$$  
+ $\lambda I(z^{1/\alpha}, z_{\min}(\phi), \tilde{z}) I(\tilde{a}, a^*) \right]^{1-\alpha} \right)^{\alpha}$

$$z_{\min}(\phi) = \frac{r \phi}{\lambda (1 - \phi)} \left( \frac{r}{\alpha} \left[ \lambda I(z^{1/\alpha}, z_{\min}(\tau_l), \tilde{z}) I(a - \kappa(\tau) a^*, a^*, \tilde{a}) + \right. \right. \right.$$  
+ $\lambda I(z^{1/\alpha}, z_{\min}(\phi), \tilde{z}) I(\tilde{a}, a^*) \right]^{1-\alpha} \right)^{\alpha}$

A similar argument to the one made in the case without corruption establishes the existence of a fixed point for both $z_{\min}(\tau_l)$ and $z_{\min}(\phi)$. 
References


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