DOES INPUT PURCHASE COOPERATION FOSTER DOWNSTREAM COLLUSION?

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Abstract

We set up a model where two retailers compete downstream and buy their inputs from a single producer. Retailers may collude downstream, when fixing the retail price and cooperate upstream by jointly negotiating the wholesale price with the producer. We find that purchase cooperation renders downstream collusion more likely. First it expands the range of differentiation where downstream collusion is a profitable strategy. Second it makes more stable the agreement downstream since the punishment becomes harsher due to the increase in the wholesale price coming from the breakdown of common upstream negotiation. The results are robust to a scenario of upstream price rigidity where the wholesale price cannot be immediately renegotiated after a deviation downstream has occurred.
1. Introduction

The concept of countervailing power, first proposed by J.K. Galbraith in 1952, as a mechanism to offset the market power exercised by large producers, has been and is still a controversial issue. Within this broad issue, the exercise of buyer power at collective level has received little scrutiny in the literature. Particularly, the question of how the purchase cooperation between competitors may affect their incentives to cooperate also—in the form of collusion—in the downstream market.

Our paper addresses this issue. We set up a simple model where two retailers interact at two stages. First, both buy a homogeneous input from an upstream manufacturer and then, retailers compete downstream selling differentiated products. There are two possibilities of cooperative behavior between retailers. They may act cooperatively when purchasing the input from the manufacturer and they may collude downstream, when setting their retail price. The vertical relationship between the producer and the retailers is modeled through a bargaining setting, where firms with market power negotiate over the linear wholesale price. In this, we follow previous work as Horn and Wolinsky (1988) and Dobson and Waterson (2007). The cooperative behavior of retailers at purchasing is represented by a common bargaining strategy, such that if there is a breakdown of negotiation with one retailer, the negotiation breaks also with the other retailer. This strategy allows the retailers to reduce the outside option of the producer at the bargaining moment, and by consequence the retailers can extract a higher surplus in the form of lower wholesale price.

Using a cooperative analysis, without looking to stability of cooperation, we find that colluding in purchasing renders collusion more likely. Starting from a non cooperative scenario, collusion in selling is only profitable for retailers if their products are homogeneous enough. However, if retailers are already colluding at purchasing, the strategy of colluding also in selling is profitable for any level of downstream differentiation. This result is explained by the countervailing effect that allow retailers to reduce wholesale price when the jointly act when negotiate with the producers. Therefore, even if purchase collusion, ceteris paribus, reduces
wholesale and retail prices, and eventually benefits final consumers, it will have an undesired effect of easing downstream collusion.

When moving to the analysis of collusion's stability, we invoke some simplifying assumptions that allow us to keep on straight track towards our target. As a first assumption we assign no strategic role to the producer. Even being a monopolist, the producer does not propose any deviation or special deal to the retailers for braking up their cooperation. Secondly, we assume that any deviation occurs at the retail market, thus leaving aside the study of purchasing deviation strategies, which have not been sufficiently analyzed until now and could certainly motivate a different paper. Finally, even though deviation occurs downstream, the punishment stage may be applied either at upstream or downstream market, depending on the collusive scheme adopted by the retailers.

Making this an infinitely repeated game and building an incentive compatibility condition over previous assumptions, we find that collusion at both stages is more stable than collusion only at selling. This means that retailers may strengthen cartel's stability by simultaneously coordinating their input purchasing as previously described. Accordingly, this initially harmless strategy may hide undesirable effects on welfare as inducing downstream collusion, inference that could not be done by exploring only price equilibriums on one-shot bargaining models.

This strong finding hinges in two facts. First colluding in both markets allows retailers to operate a stronger punishment after a deviation. The punishment strategy does not only imply that retailers will return to the static game level downstream. Also it will lead to the break of the joint purchasing agreement that allowed distributors to enjoy lower wholesale prices. Thus, by breaking cooperation at both stages of the vertical chain, firms may apply a harsher punishment, that certainly reinforces the stability of the cartel.

The second effect comes from the difference in changes of wholesale prices in the punishment phase. Even if the deviation does not break the joint purchase agreement, the punishment is more harmful when distributors are colluding in
both stages. If they collude only in selling, the punishment will be smoothed because the new wholesale price will be reduced due to lower rents available downstream. However, if they collude in both stages, the punishment applied only downstream, leads to an increase in the wholesale price, which decrease further distributors profits at the punishment phase. This difference in the direction of the wholesale price change that occurs after a deviation is explained by the type of negotiation between distributors and the producer.

Our results are robust to the incorporation of wholesale prices’ rigidity into the model. We acknowledge that input purchasing contracts are often fixed on a longer term that retail prices. For instance, supermarkets are able to change their retail prices almost instantly but wholesale prices are much more difficult to modify because they are the outcome of a negotiation between parts. We would expect that upstream price rigidity could limit the ability to apply punishment at the purchase stage. Nevertheless, we find that still collusion at both stages is more stable than collusion only at selling. In the extreme case of total wholesale price rigidity, the first effect described above is lost, but the second effect still holds.

The findings of this article dispute the beneficial impact alleged by the countervailing power theory. In a framework where collaboration is also feasible at the downstream market, the exercise of collective buyer power reinforces the stability of a collusive agreement, which yields to higher retail prices, hurting final consumers.

Our work has two additional contributions. First, we model purchase agreements in a different way as has been treated in the literature so far. By using the parallel disagreement rather than the joint maximization of benefits in the negotiation, we get that coordination in the purchase will always be a profitable strategy for retailers. Second, we incorporate the asymmetry between the pricing upstream and downstream. The wholesale price, which is negotiated among companies with market power and set out in contracts, has greater rigidity than retail prices, which can be changed easily. Our results show that even in a scenario of extreme rigidity of wholesale prices, coordination in the purchase is still useful to facilitate
collusion in the sale. The difficulty in changing wholesale prices reduces the intensity of the punishment strategy and therefore is a stricter scenario where collusion may occur.

**Relation to the Literature.**

In a static setting, our results support the countervailing power hypothesis, because purchase cooperation pushes down wholesale prices and some of that reduction are passed through consumers in form of lower retail prices. Nevertheless it ceases to exist when we set a dynamic scenario for competition and cooperation, where retailers can exploit not only their buyer’s but their seller’s power as well. This was already intuited by Stigler (1954) and Hunter (1958), who claimed that powerful buyers may not pass their cost reductions to final consumers, and then demonstrated by Von Ungern-Sternberg (1996) in a quantity setting model and by Dobson & Waterson (1997) in a price setting model. Our work differs from the latter two in that these authors depict a gain in purchasing power by increasing the concentration in the retail segment of the industry.

On the other hand, Chae & Heidhues (2003) reviewed buyer’s alliances and their welfare effects through a model were two buyers from independent markets pool their input purchase and alternate the negotiation with the supplier. This is, therefore, a source of buyer’s power similar to which we are considering here, in terms that it does not disturb downstream equilibrium strengthening seller’s power at a time. Thereby, risk aversion is here a sufficient condition for making buyer’s alliances desirable. However, what we think is the main difference to our work is that Chae & Heidhues (2003) leave aside any possibility of downstream coordination ones setting independent market for buyers. This is also what we consider an important contribution of our own work since in most cases firms that compete purchasing an input face each other again while selling. Moreover, Lustgarten (1975) showed, by means of US data analysis, that buyer’s power is positively correlated to seller’s concentration within industries, which suggests that studying buyer’s power in isolation involves overlooking downstream behavior among firms that may completely shift any welfare conclusions.
Finally, we would like to relate our work to Horn and Wollinsky (1988). The authors show that retailers will be worse off by merging when their products are substitutes, but this is intuitively opposite to the results we get under purchasing and selling simultaneous collusion, which is always the most profitable strategy for retailers despite their products being substitutes. Regarding this discordance, we highlight that the only difference between a merger and the way we model collusion at both stages, resides in the purchasing cooperation scheme adopted. In fact, retailers still negotiate separately when they cooperate upstream, which does not happen under a merger.

This article is organized as follows: Firstly we develop our model and derive upstream and downstream price equilibriums. We compare prices and profits under competition and under each kind of cooperative strategy here considered. Secondly, we build incentives’ compatibility restrictions and compare stability of selling collusion and collusion at both stages. Then we repeat this exercise under different levels of wholesale price rigidity. Finally, some concluding remarks are presented to close this paper.
2. The Model

We characterize the industry as follows: There are two distributors: $D_1$ and $D_2$, who buy a homogenous input from a manufacturing firm $M$. The distribution services create differentiation in the downstream market. Each product has an inverse demand function of the form $p_i = 1 - q_i - \beta q_j$, with $i = 1, 2$, where $\beta \in [0, 1)$ is the parameter that represents the differentiation of final goods. Thus, when $\beta = 0$ the goods are completely independent from the point of view of consumer preferences, while when $\beta \to 1$, they become perfect substitutes. If we reverse these functions, we obtain the direct demands, given by $q_i = \frac{1-\beta - p_i + \beta p_j}{1-\beta^2}$. Marginal costs of $M$ are normalized to zero and invariant to changes in the level of production. The marginal costs of distributors, apart from the wholesale price, are identical and equal to zero as well.

We assume that both the producer and the distributors have bargaining power over the wholesale price. This may correspond to the case of a well-recognized brand by consumers, whose products are sold through two large retail chains that differentially compete for sales to final customers. Linear wholesale prices are drawn from two separate and simultaneous negotiations between each distributor and the producer. Following the literature, - Horn & Wollinsky (1988) and Dobson & Waterson (1997) - we model the vertical negotiations over linear wholesale prices using the Nash bargaining solution. Additionally, in order to simplify the modeling and focusing on the effects of collusion over negotiations’ outcome, we assume that there is equal bargaining power between firms.

We start by modeling the base case (i) in which retailers adopt a non-cooperative behavior, both on input purchase and on selling. Then we analyze the cooperative scenarios. These are: (ii) collusion over input purchasing, (iii) collusion over final goods’ selling and (iv) collusion at two stages: purchase and sale. At the end of the section we compare the equilibrium prices and benefits of the four scenarios examined.
(i) Non-cooperative game between distributors

The sequence of decisions in this scenario is as follows: At $t=1$, the producer $M$ simultaneously negotiate with distributors $D_1$ and $D_2$. As a result of the negotiation, the wholesale price $w_i$ emerges for each retailer. At $t = 2$, downstream distributors compete on prices, knowing the wholesale prices set at $t = 1$ for both firms.

Once the wholesale prices have been set at $t=1$, each distributor at $t=2$ maximizes its profits, following a non-cooperative strategy.

$$\max_{p_i} \pi_{D_i} : (p_i - w_i)q_i(p)$$

(1)

Where $q_i(p)$ is the demand function of good $i$, and $p = (p_i, p_j)$ is the downstream price vector. The first order condition of this maximization problem is:

$$\frac{\partial \pi_{D_i}}{\partial p_i} : 1 - \beta - 2p_i + \beta p_j + w_i = 0$$

Then, we obtain the equilibrium retail or downstream price of each good as a function of the wholesale price vector and the parameter of the demand function:

$$p_i(w) = \frac{(1 - \beta)(2 + \beta) + 2w_i + \beta w_j}{4 - \beta^2}$$

Anticipating downstream equilibrium, the problem over which the producer and each distributor negotiate in $t = 1$ is represented by:

$$\max_{w_i} \pi_{D_i}(w)[\pi_M(w) - \Delta_M(w_j)]$$

(2)

Where $\Delta_M(w_j) = w_j z_j(w_j)$, are the outside option payoffs of the producer. These are the benefits that $M$ would obtain if the negotiation breaks down with $D_i$ and $M$ sells its product only to $D_j$, at a price $w_j$. Since in the downstream market $D_j$ will have no competitor, its demand function will be equivalent to the one of a monopolist: $z_j(w_j) = \frac{1-w_j}{2}$. We further assume that the outside payoffs of each distributor are zero, because there are no other producers providing the same input. Finally, we denote the benefits of each part –distributors and producer- as
\[ \pi_{D_i}(w) = (p_i(w) - w_i)q_i \] and \[ \pi_M(w) = w_iq_i(w) + w_jq_j(w), \] where \( w = (w_i, w_j) \) is the wholesale price vector.

The first order condition of the negotiation problem at \( t = 1 \) is:

\[ \frac{\partial \pi_{D_i}}{\partial w_i}[\pi_M - \Delta_M] + \frac{\partial \pi_M}{\partial w_i}\pi_{D_i} = 0 \]  

(3)

We obtain the symmetric equilibrium at the negotiation stage on wholesale prices:

\[ w^N = \frac{(2 + \beta)(1 - \beta)}{(4 - \beta^2)(2 - \beta) - \beta^4} \]  

(4)

Where the upper index N stands for Nash or Non-cooperative equilibrium.

Downstream prices and firms' profits become:

\[ p^N = \frac{1 - \beta + w^N}{2 - \beta} \quad q^N = \frac{1 - w^N}{(1 + \beta)(2 - \beta)} \]  

\[ \pi^N_{D_i} = \frac{(1 - \beta)(1 - w^N)^2}{(1 + \beta)(2 - \beta)^2} \quad \pi^N_M = \frac{2w^N(1 - w^N)}{(1 + \beta)(2 - \beta)} \]  

(5)

(6)

We summarize the behavior of prices in function of exogenous parameter \( \beta \) in the following lemma:

**Lemma 1:** If two distributors compete downstream in prices and negotiate the wholesale price of the input with a single producer, the equilibrium prices are characterized by: (i) wholesale and retail prices decreasing in the degree of homogeneity of distribution services \( \beta \), (ii) both prices converging to zero when the services become close to perfect substitutes.

The result of the impact of differentiation on downstream prices is not surprising. However, the effect of differentiation on wholesale prices is not trivial. As we observe from equation 5, when downstream goods become perfect substitutes, the wholesale price is equal to zero and the upstream firm M is not able to get any profit even if it is a monopoly. The reason behind this counterintuitive result is that in a scenario of simultaneous negotiation con the two retailers, M is not capable to commit to a higher whole price. For any price \( w_i \geq 0 \), M will always have incentive to undercut this price through the wholesale price \( w_j \) in the
negotiation con Dj. As a result, no price above marginal cost will be credible to sustain in the extreme case of perfect substitution downstream

(ii) Cooperation in Purchasing

We now release the assumption that retailers act non-cooperatively in the purchase of the input. In this section, we allow retailers to coordinate their actions at the moment of negotiation with the producer M. To do this, we represent the game through the following timing:

\( t = 0 \): The distributors decide whether to use a cooperative strategy or non-cooperative in the purchase. If they choose to cooperate, each distributor threats the manufacturer with not to purchase the input, if the negotiation breaks down with the other distributor.
\( t = 1 \): Two simultaneous negotiations take place, one with each distributor. As a result, wholesale prices are obtained.
\( t = 2 \): Distributors compete in downstream prices if in \( t = 1 \) both reach an agreement with the producer.

Notice that we define purchasing collusion as the commitment of both retailers not to buy from the producer in case of a disagreement with one of them. This is different from the literature of downstream mergers, where firms jointly negotiate the wholesale price with the manufacturers.

The problem to solve in every negotiation continues to be represented by (2) and the maximization problem of each downstream dealer is still (1). The only relevant change is that, given the coordinated action between retailers when buying the input, the benefits of disagreement of the producer are zero, i.e.: \( \Delta_M = 0 \).

The symmetric wholesale price equilibrium, given the joint upstream negotiation and downstream competition is the following:
Where the upper index B stands for collusion at the buying stage. Since downstream competition does not change but the new wholesale price does it, we can employ equations (5) and (6) to obtain the new equilibrium. We summarize the result of this section in the following lemma:

**Lemma 2**: When the retailers cooperate at negotiating with the producer, the equilibrium is characterized by: (i) wholesale and retail prices decreasing in the degree of homogeneity of distribution services $\beta$, (ii) both prices converge to zero when the services are close to perfect substitutes.

**(iii) Collusion in Selling**

The timing of this new game is similar to the previous one, but instead of cooperating in the upstream negotiation, distributors decide to collude in the downstream market. The producer knows that collusion exists downstream. The wholesale price negotiation is done individually, and at $t = 2$ retailers set prices that maximize their joint profits: $\pi_{D_i} + \pi_{D_j} = (p_i - w_i)q_i(p) + (p_j - w_j)q_j(p)$. These joint profits are optimized at the monopoly price $p_i(w_i) = \frac{1+w_i}{2}$.

After negotiating at $t = 1$, the resulting wholesale price under downstream collusion becomes:

$$w^S = \frac{1}{4-\beta}$$

(8)

Where the superscript S denotes the result of collusion in the sale activity. In equilibrium, downstream prices, quantities and distributors benefits are the following:

$$p^S = \frac{1+w^S}{2}$$

$$q^S = \frac{1-w^S}{2(1+\beta)}$$

$$\pi^S_{D_i} = \frac{(1-w^S)^2}{4(1+\beta)}$$

$$\pi^S_M = \frac{w^S(1-w^S)}{(1+\beta)}$$

(9)  
(10)
We characterize this new equilibrium in the following Lemma:

**Lemma 3:** When retailers negotiate the wholesale price individually but collude in the downstream market, the equilibrium is characterized by: (i) wholesale and retail prices are increasing in the degree of homogeneity of distribution services $\beta$ (ii) When downstream products tend to perfect substitutes, the wholesale price converges to $1/3$ and the retail price converges to $2/3$.

Unlike the two previous scenarios, the wholesale price resulting from negotiations with colluding distributors is increasing in $\beta$. This is so because the producer has fewer incentives to undercut wholesale prices, since diminished downstream competition implies that a lower distributor’s cost will be passed to final consumers in a minor proportion, i.e., $\frac{\partial p^*_i(w)}{\partial w_i} \leq \frac{\partial p^N_i(w)}{\partial w_i}$. On the contrary, most of the cost savings will rather enlarge distributors’ surplus at manufacturer’s own expense. Hence, with lower undercutting incentives, higher wholesale prices become more credible to sustain for the manufacturer. The second part of the explanation comes from the fact that M will now be able to better exploit his bargaining position as the distribution services become closer substitutes, since he can substitute at a lower cost each channel of distribution. This explains us why wholesale prices are not only non-decreasing but even increasing in the parameter of product differentiation under selling collusion.

**(iv) Collusion in Purchasing and Selling**

The timing of the game when distributors act cooperatively at the purchasing and selling stage is the following:

$t=0$: The distributors decide whether to collude or not simultaneously in both markets. As in section (ii) if they collude, each distributor commit not to buy to M if the negotiations breaks down with the other one.

$t=1$: Two simultaneous negotiations about the wholesale price occur between the producer and each distributor.

$t = 2$: If both distributors reach an agreement in wholesale price with the producer, they collude downstream, setting the price that maximizes their joint profits.
The problem to be solved by the cartel at $t = 2$ is the same as in section (iii) when retailers collude in the sale activity. Moreover, since the purchase collusion persists, we set $\Delta_M = 0$.

Based in previous results, the bargaining results at $T = 1$, boils down to:

$$w^{BS} = \frac{1 - \beta}{2(3 - 2\beta)} \quad (11)$$

We characterize this new equilibrium in the following lemma:

**Lemma 4:** When retailers collude simultaneously at the purchasing and selling stage, the equilibrium is characterized by: (i) The wholesale and retail prices are decreasing in the degree of homogeneity of distribution services ($\beta$), (ii) The wholesale price converges to 0 and the retail price converged to 1/2 when the services are close to perfect substitutes.
2.1 Comparative Analysis

Table 1 summarizes the equilibrium prices in the four scenarios analyzed. Here, we present the analytical form of wholesale and retail price under each of the four strategies adopted by distributors in the bargaining model. Figure 1 shows simulations of the wholesale and retail prices for different values of \( \beta \), illustrating the messages of lemmas 1 to 4.

Table 1: Equilibrium Prices

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( T = 1 )</th>
<th>( T = 2 )</th>
<th>( w )</th>
<th>( p )</th>
</tr>
</thead>
</table>
| Nash           | \(-\) \[
\max_{p_i} \pi_{D_i} \]
(2 + \( \beta \))(1 - \( \beta \))
\left[ (4 - \beta^2)(2 - \beta) - \beta^4 \right]
\left[ (2 + \beta)(1 - \beta) \right]
\left[ 2(6 - \beta - 3\beta^2) \right]
\frac{1 - \beta + w^N}{2 - \beta} |
| Purchase       | \( \Delta_M = 0 \) \[
\max_{p_i} \pi_{D_i} \]
\frac{1}{4 - \beta}
\frac{1 + w^V}{2} |
| Collusion      | \( \Delta_M = 0 \) \[
\max_{p_i} \pi_{D_i} + \pi_{D_j} \]
\frac{1 - \beta}{2(3 - 2\beta)}
\frac{1 + w^{CV}}{2} |
| Selling        |                      |             |         |       |
| Collusion      |                       |             |         |       |
| Both           | \( \Delta_M = 0 \) \[
\max_{p_i} \pi_{D_i} + \pi_{D_j} \]
\frac{1 - \beta}{2(3 - 2\beta)}
\frac{1 + w^{CV}}{2} |

We emphasize, firstly, that both prices are always lower under collusion in purchase than under competitive equilibrium, demonstrating the effectiveness of this type of cooperation to reduce wholesale prices. Moreover, in this model, part of this cost saving reaches the hands of consumers in form of lower retail prices, since purchase in cooperation just reduces the input price for the competing distributors.
The most relevant results of the comparative analysis on both prices are summarized in the following proposition:

**Proposition 1:** (i) The collusion on purchase reduces both wholesale and retail prices for any strategy adopted at selling. (ii) Selling collusion raises not only the retail price, but also the wholesale price. Additionally, both prices are higher under selling collusion than in any of other three scenarios. (iii) The simultaneous cooperation on purchasing and selling reduces the retail price, with respect to the case of no collusion in both stages, only if the downstream products are sufficiently differentiated, i.e. $\beta < 0.22172$.

Proof: See the Appendix.

We continue with the comparison of the benefits that distributors obtain under each of the four strategies previously examined. Accordingly, we present in Figure 2 the simulations. As expected, the most profitable strategy for retailers is when they collude simultaneously both at the purchase and the sale stage. Colluding at both activities allows distributors to enjoy the benefits of reduced competition downstream, without sharing the rents with the producer. It is also clear that colluding in the purchase activity is a profitable strategy, whether in the downstream market there is competition or collusion.
Analyzing the strategy of collusion in selling, it will always be profitable to adopt this strategy if retailers are colluding in the purchase at the same time. However, when firms are initially in a scenario of no cooperation (Nash), collusion in the sale is not always a dominant strategy. As shown in the graph 2, the benefits of acting non-cooperatively are greater than those of colluding in selling only to high enough levels of differentiation. This is case when $\beta \geq 0.65092$. This result is explained by two effects that affect the profits of the distributors when decide to collude downstream. First, for any wholesale price, collusion in selling increases the profits of retailers. But there is a second effect, the wholesale price increases due to the stage of negotiation with the producer. The latter, knowing that there are more rents downstream, will ask for a greater participation on them, by the way of requiring a higher wholesale price.

As demonstrated in Proposition 1, the wholesale price increases when downstream moves to collude from a non-cooperative behavior. This second effect is opposite of the first, since it reduces the profits of distributors. As a result we will have that for higher levels of homogeneity of the final product, the first effect dominates the second and therefore the benefits of distributors increases. Conversely, if retailers are heterogeneous, the second effect dominates the first, and rents will fall as a consequence of colluding in sale. The main results regarding the incentives for distributors to engage in any collusive strategy is summarized the following proposition:
**Proposition 2:** (i) Distributors will always have incentives to collude in the purchase, regardless of the strategy they follow in the sale. (ii) If there is collusion in the purchase, there will be incentives to collude in the sale too. (iii) Starting from a scenario of no collusion, it will profitable to collude in selling if and only if the downstream services are sufficiently homogeneous, i.e., if \( \beta \geq 0.65092 \).

Proof: See the appendix.

3. **Stability of Collusion**

In the previous section we analyzed, from a cooperative point of view, the incentives of distributors to collude either at the purchase or sale activity. In this section, we examine the collusion from a non-cooperative point of view. It means, we look whether cooperation is a sustainable strategy in the long term, in a scenario where firms interact repeatedly over time. There are some particularities in the problem under analysis that make it different from the standard case where two sellers collude. In our model, cooperation occurs in two activities: purchasing and selling. The special complexity to the analysis of stability is the interaction between these two actions of collusion. Particularly in the deviation and punishment strategies that companies can apply.

Given the results of the previous section, we now seek to clarify whether collusion at purchase, besides raising the profitability of collusion in the sale, facilitates the stability of the non-cooperative strategy in the repeated game that retailers play. To do this, we consider the same strategies as before: either collusion at purchase or sale, but endlessly repeated. For each possible cooperative strategy, we formulate the incentive compatibility constraint.

The main assumptions of the repeated game with the possibility of collusion in sale and purchase are:

**Deviation occurs only at selling.**

The producer is completely passive and cannot act jointly with one of the
distributors in order to improve their profits to the detriment of the other retailer. This assumption may be a strong limitation on the actions the producer may undertake. Nevertheless, we believe that there is no an obvious way to model the deviation of a retailer when dealing with an upstream producer. Unlike what happens downstream, where the product is sold to atomized consumers, the interaction upstream occurs with a single producer. The modeling of possible deviations in the relationship with the producer is a task that is beyond the objectives set out in this article.

On the other hand, assuming a passive buyer imposes a stricter scenario for collusion. Firms cannot employ the wholesale price as an instrument to enable collusion downstream. The wholesale price will be the outcome of a negotiation that will depend on whether retailers collude upstream or downstream, and it cannot be modified in a way affects the likelihood of collusion.

**No wholesale price renegotiation at deviation.**

Deviation is an unexpected action for the other party, therefore the wholesale price negotiated between the producer and both retailers do not change, when one distributor decides to sell more in the downstream market. Consequently, the distributor that deviates upstream simply buys a different amount of quantity but at the same wholesale price agreed. Note that the agreement between each distributor and the producer is based on a fixed wholesale price per unit, regardless of the quantity purchased.

**Punishment Strategy**

Depending of the extent of collusion, the punishment strategy can be applied at the upstream market, at the downstream market or both. We employ a grim punishment strategy. If there is collusion only at selling, then the punishment, following a deviation leads firms to play non-cooperatively in the downstream market for the remaining periods. If firms are colluding at both stages, punishment can be applied either at downstream market or both. In the latter case, distributors remove their purchasing cooperation and return to negotiate separately the
wholesale price with the producer, without linking their success or failure in collusion when dealing with the producer.

**Upstream Price Rigidity**

If the punishment strategy is applied upstream, retailers will break the agreement of joint negotiation with the manufacturer, and the wholesale price will be obtained through individual negotiations instead. However, breaking the purchase agreement might not be immediate. Usually the wholesale price has greater rigidity than the retail price. Since the former is the result of costly negotiations, it is held constant for some span of time, normally one year. By contrast, distributors such as supermarket chains, change prices of items with higher frequency. Wholesale prices are set in contracts that are valid for some time duration, while in retailing, we have spot price, where consumers decide whether to buy or not according to the price offered.

In order to deal with this asymmetry in price rigidity that affects the timing of punishment, we will start assuming that upstream renegotiation is immediate. It means that there is full flexibility to change wholesale prices when competition downstream has been modified. Then, we move to the scenario of wholesale price rigidity, such that the punishment, that involves price renegotiation will have to wait until current wholesale prices expire.

### 3.1 Wholesale Price Flexibility.

For any collusive strategy, we define the retail deviation price as:

\[
p^d = \arg\max_{p_i} \left( (p_i - w^c) \left( \frac{1 - \beta - p_i + \beta (1 + w^c)/2}{1 - \beta^2} \right) \right) = \frac{2 - \beta + (2 + \beta)w^c}{4} \tag{12}
\]

Where \( w^c \in \{w^S, w^{BS}\} \) is the set of wholesale prices under retailers’ cooperation and \( p^c = p^c = \frac{1+w^c}{2} \). For the sake of simplicity, in what follows we omit the lower index D on distributors profit \( \pi \).

One-period deviation profits are \( \pi^d (p^d, p^c) \) with \( \pi^d \in \{\pi^{dS}, \pi^{dB}\} \). Accordingly, we define \( \pi^c \in \{\pi^S, \pi^{BS}\} \) as the collusive profits of distributors. Further, \( \pi^p \in \)
\( \{\pi^N, \pi^B\} \) corresponds to the punishment payoffs, which are equal to non-cooperative profits either when selling collusion is broken or when collusion at both stages is dismantled.

Therefore, under grim punishment strategy, collusion will dominate deviation if and only if:

\[
\sum_{t=1}^{\infty} \delta^t \pi^c \geq \pi^d + \sum_{t=2}^{\infty} \delta^t \pi^p
\]

(13)

Where \( \delta \in (0,1) \) is the discount factor which we have assumed equal and constant for \( D_1 \) and \( D_2 \). We can simplify the above expression by the following condition:

\[
\frac{1}{1-\delta} \pi^c \geq \pi^d + \frac{\delta}{1-\delta} \pi^p
\]

This condition implicitly defines a critical value \( \delta^c \in \{\delta^S, \delta^B_s, \delta^B_s\} \) for the discount factor under selling collusion (\( \delta^S \)) and under collusion at both stages with two different punishment strategies \( \delta^B_s, \delta^B_s \). Thus collusion will be stable over time if the parameter \( \delta \) satisfies:

\[
\delta \geq \delta^c = \frac{\pi^d - \pi^c}{\pi^d - \pi^p}
\]

(14)

In the case of collusion only in selling, the critical value \( \delta^S \) corresponds to:

\[
\delta^S = \frac{\pi^dS - \pi^S}{\pi^dS - \pi^N} = \frac{\beta^2(2-\beta)^2(1-w^S)^2}{(2-\beta)^4(1-w^S)^2 - 16(1-\beta)^2(1-w^N)^2}
\]

(15)

For collusion at both selling and purchasing but punishment only in selling:

\[
\delta^B_s = \frac{\pi^dBS - \pi^BS}{\pi^dBS - \pi^B} = \frac{\beta^2(2-\beta)^2(1-w^{BS})^2}{(2-\beta)^4(1-w^{BS})^2 - 16(1-\beta)^2(1-w^B)^2}
\]

(16)

Whereas collusion at both selling and purchasing and punishment in both activities we have:

\[
\delta^B_{bs} = \frac{\pi^dBS - \pi^BS}{\pi^dBS - \pi^N} = \frac{\beta^2(2-\beta)^2(1-w^{BS})^2}{(2-\beta)^4(1-w^{BS})^2 - 16(1-\beta)^2(1-w^N)^2}
\]

In graph 3 we simulate the critical discount factor in function of the differentiation parameter \( \beta \), for three possible collusive schemes.
In order to facilitate the comparison between the discount factors corresponding to the three different collusive and punishment strategies, we first present an intermediate result.

**Lemma 5.** The critical discount factor $\delta^c$, above which collusion is feasible, is independent of the wholesale price that retailers face, as long as the wholesale price is the same in the cooperative and in the deviation and punishment phase. Moreover, if all possible wholesale prices are equal, then the tree critical discount factors are equal as well.

Proof:

The demonstration follows straightforward from equations (15), (16) and (17), when fixing $w^c = w^d = w^p$ in each of them. We define as $\delta$, the wholesale price invariant critical discount factor, which can be obtained through any of the above equations, yielding to

$$
\delta = \frac{1}{1 + 4(1 - \beta)/(2 - \beta)^2}
$$
Now, we move to the following proposition:

**Proposition 3:** For any level of downstream product differentiation, collusion at both stages: purchasing and selling is easier to sustain than collusion only at selling, no matter the punishment scheme that firms agree to follow. Furthermore, we have that: $\delta^S \geq \delta^{BS}_S \geq \delta^{BS}_{BS}$.

Proof:
First, we will show the first inequality. Since $w^S \geq w^N$ then: $\pi^N(w^N) \geq \pi^N(w^S)$, which implies: $\delta^S \geq \hat{\delta}$. Since $w^B \geq w^{BS}$ then: $\pi^N(w^{BS}) \geq \pi^N(w^B)$, and by consequence: $\hat{\delta} \geq \delta^{BS}_S$. Therefore, we obtain that: $\delta^S \geq \hat{\delta} \geq \delta^{BS}_S$.

On the second inequality, the only term that differs between the critical discount factors $\delta^{BS}_S$ and $\delta^{CV}_{CV}$ is the non-cooperative profits of the punishment phase. If the punishment strategy is only at selling, the wholesale price is $w^C$. On the contrary, if the punishment is at both stages, then the wholesale price turns in $w^N$. Since $w^B \leq w^N$, then: $\pi^N(w^N) \leq \pi^N(w^B)$, or equivalently: $\pi^N \leq \pi^B$, which implies that: $\delta^{BS}_S \geq \delta^{BS}_{BS}$.

Cooperation at buying renders more likely collusion at selling as well. This result hinges in two effects, each one represented by one of the inequalities of the proposition 3. The second inequality says that collusion is more sustainable when the punishment is applied to both stages, purchasing and selling. Indeed, punishing upstream and downstream is more costly for firms, since not only brings downs downstream profit to the Nash level, but also increases wholesale price, which reduce further distributors profits at the punishment phase. Including in the punishment strategy the break of the upstream negotiation has a similar effect on the sustainability of collusion as the multimarket contact model.

The first inequality of proposition 3 is more complex to understand, but following the steps of the proof it is easier to get the intuition behind it. When firms collude only at selling, the Nash reversion stage following a deviation, is mitigated due to the change in the wholesale price. In fact, upstream negotiation under a non-
cooperative game downstream reduces the wholesale price, compared with the case when firms collude downstream. This reduction in the input price renders the punishment softer, and by consequence collusion is more difficult to sustain. The opposite effect occurs when firms are simultaneously colluding upstream and downstream. If one firm deviates, the wholesale price goes up after the new negotiation with the producer, which reduces the profits of firms in the punishment phase, strengthening the collusive agreement between retailers. This change in the direction of non cooperative profits following a renegotiation is explained by the fact that: \( w^S \geq w^N \) and \( w^{BS} \leq w^B \).

Proposition 3 contains the main result of the article. When firms agree to collude at both stages, it is easier to sustain the cooperative agreement, respect to the case when firms collude only at selling. The implications of this result are relevant, since by acting cooperatively at purchasing an input, firms are facilitating themselves to collude at the retail level. So, the supposedly harmless joint purchasing agreement may hide and undesirable effect on welfare, which is inducing the collusion downstream.
3.2 Wholesale Price Rigidity

In this section we analyze the case where wholesale prices cannot be renegotiated immediately after a deviation have occurred. We assume that retail prices can be changed every period, but wholesale negotiation takes place every $T$ periods. For the three possible collusive strategies, we calculate the deviation value $\Phi$ in function of $X$. Where $X$ is defined as the remaining time –in periods- before a new upstream negotiation takes place, such that $X \in [1, T]$

Generalizing for any collusive strategy and any moment in time, the present value of deviating from the agreement, $\Phi \in \{\Phi^S, \Phi^{RS}_S, \Phi^{RS}_{DS}\}$, is obtained from:

$$\Phi = \pi^d(w^C) + \sum_{t=1}^{X} \delta^t \pi^p(w^C) + \sum_{t=X+1}^{\infty} \delta^t \pi^N(w^p)$$  \hspace{1cm} (18)

For the following periods after the deviation, the punishment takes place with the already negotiated wholesale price $w^C$. Then, after the next negotiation occurs, the wholesale price goes down to $w^p$.

Equation 18 can be expressed as:

$$\Phi(X) = \pi^d + \frac{\delta}{1 - \delta} \pi^p(w^C) + \frac{\delta^X}{1 - \delta} [\pi^p - \pi^p(w^C)]$$  \hspace{1cm} (19)

Where $\pi^d = \pi^d(w^C)$ and $\pi^p = \pi^p(w^p)$.

In order to verify if the incentive compatibility condition is satisfied for sustaining collusion, we need to evaluate the deviation value $\Phi(X)$ at its maximum value.

**Lemma 6:** When distributors collude only at selling, the value of the deviating strategy is decreasing in the remaining time to negotiate again with the producer. On the contrary, when distributors collude both at selling and purchasing, and the punishment is either only at selling or selling and purchasing, the deviation value is increasing in the remaining time to the next upstream negotiation.
Proof:
Given that \( \frac{d\phi(X)}{dX} = \frac{\delta^X}{1-\delta} \ln(\delta) \left[ \pi^P - \pi^P(w^c) \right] \), and considering that \( \ln(\delta) \leq 0 \), \( \phi(X) \) will be increasing in \( X \) if and only if \( \pi^P - \pi^P(w^c) \leq 0 \), and decreasing otherwise. For collusion at selling, \( \pi^P = \pi^N \) and \( w^c = w^S \), so:
\[
\pi^N > \pi^N(w^S)
\]
and thus \( \frac{d\phi^S(X)}{dX} \leq 0 \). Similarly, we will have \( \frac{d\phi^S(X)}{dX} \geq 0 \) and \( \frac{d\phi^S(X)}{dX} \geq 0 \) due to the fact that \( \pi^C < \pi^C(w^BS) \) and \( \pi^N < \pi^C(w^BS) \).

Lemma 6 has the following implications. In the case of collusion only at selling, the value of deviating \( \phi \) is maximized at \( X = 1 \), which is just one period before the next upstream negotiation takes place. On the other hand, the value of cooperating is independent of \( X \) because the next upstream negotiation does not alter the value of profits in every period. Therefore, in order to verify if collusion is sustainable, the relevant incentive compatible constraint to satisfy is the one evaluated at \( X = 1 \). If collusion is sustainable at \( X = 1 \), then is sustainable at any time between upstream negotiations. On the contrary, if the value of deviating for \( X = 1 \) is greater than the value of cooperating, then collusion is not sustainable. Note that in the latter case, deviation would take place immediately after the upstream negotiation has finished. Retailers anticipating that deviation will occur later will not be willing to follow a cooperative strategy at any time. Consequently, when collusion is only at selling the critical discount factor above which collusion is feasible is the same as we calculate through equation (4), i.e. \( \delta^S \). Since the relevant incentive compatible constraint is the one evaluated at \( X = 1 \), price rigidity upstream plays no role in the feasibility of collusion only in selling.

The results for case of collusion at both stages are different. Since the value of deviation is increasing in \( X \), firms will have the higher incentive to deviate just the next period after the upstream negotiation occurred. The relevant constraints, in these cases, are those evaluated at the maximum value of \( X \), i.e. at \( X = T \). If the incentive compatible constraint to keep cooperating, is satisfied for \( X = T \), then is also satisfied for any \( X < T \). As in the case of only selling collusion, the value of cooperating is independent of the time when the strategy is evaluated.
The critical discount factor for both upstream collusive strategies are obtained from the following equations.

\[ \frac{\pi^{BS}}{1-\delta} = \pi^{dBS} + \frac{\delta}{1-\delta} \pi^{C}(w^{BS}) + \frac{\delta^T}{1-\delta} [\pi^{C} - \pi^{C}(w^{BS})] \]  \hspace{1cm} (21)  

\[ \frac{\pi^{BS}}{1-\delta} = \pi^{dBS} + \frac{\delta}{1-\delta} \pi^{C}(w^{BS}) + \frac{\delta^T}{1-\delta} [\pi^{N} - \pi^{N}(w^{BS})] \]  \hspace{1cm} (22)  

Rearranging terms on equation 21, we obtain:

\[ \delta [\pi^{dBS} - \pi^{C}(w^{BS})] + \delta^T [\pi^{C}(w^{BS}) - \pi^{C}] + \pi^{dBS} - \pi^{BS} = 0 \]  \hspace{1cm} (23)  

Taking the total derivative on equation (23), yields to:

\[ \frac{d\delta}{dT} = \frac{-\delta^T \ln(\delta) [\pi^{C}(w^{BS}) - \pi^{C}]}{[\pi^{dBS} - \pi^{C}(w^{BS})] + T \delta^{T-1} [\pi^{C}(w^{BS}) - \pi^{C}]} \geq 0 \]  

The above derivative is positive, since: \( \delta \leq 1 \Rightarrow \ln(\delta) \leq 0; \pi^{C}(w^{BS}) \geq \pi^{C} \) and: \( \pi^{dBS} \geq \pi^{C}(w^{BS}) \). It means that the critical discount factor, above which collusion is feasible, is increasing in T, the remaining time to the next wholesale price negotiation.\(^3\) The intuition of this result is that part of the punishment for a deviation – the increase in the wholesale price – is delayed until next input price negotiation takes place. Moreover, in the extreme case of no new wholesale price negotiation, i.e. \( T \to +\infty \), the upstream punishment effect totally disappears.

---

\(^3\) The same analysis applies if we take the derivative in equation (22) since: \( \pi^{N}(w^{BS}) \geq \pi^{N} \).
**Proposition 4.** When $T \to +\infty$ and distributors simultaneously collude in selling and purchasing, punishing a deviation only in the downstream market is equivalent, in terms of incentives, to punishing through both downstream and upstream markets.

(ii) Collusion at purchasing and selling is easier to sustain than collusion only at selling, no matter the level of wholesale price rigidity represented by $T$.

Proof: (i) Taking the limits of $T \to +\infty$ in equations 21 and 22, the third term of the right hand side of both equations disappears and both equations become equivalent. The common critical discount factor when $T \to +\infty$ is:

$$
\lim_{T \to \infty} \delta^{BS} = \frac{\pi^{dBS} - \pi^{BS}}{\pi^{dBS} - \pi^{N}(w^{BS})}
$$

Proof (ii) From lemma 5, we have that: $\lim_{T \to \infty} \delta^{BS} = \hat{\delta}$. In fact, the critical discount factor obtained in the above proof is equivalent to the wholesale price invariant discount factor $\hat{\delta}$, since the wholesale price is constant along the three phases and is equal to: $w^{BS}$. 

**Graph 4:** Critical discount factor in function of $\beta$, for different periods $T$. 

![Graph 4](image)
If retailers collude only at selling, the critical discount factor $\delta^S$ is equal to:

$$\delta^S = \frac{\pi^{dS} - \pi^S}{\pi^{dS} - \pi^N}$$

Then, we obtain that $\delta \leq \delta^S$ since $\frac{\pi^{dS} - \pi^S}{\pi^{dS} - \pi^N(w^S)} \leq \frac{\pi^{dS} - \pi^S}{\pi^{dS} - \pi^N(w^N)}$ because: $\pi^N(w^N) \geq \pi^N(w^S)$. Therefore, we have that: $\lim_{T \to \infty} \delta^{BS} \leq \delta^S$, which proves the proposition.

The first result of proposition 4 says that collusion in both stages is equally feasible, in terms of stability, whether distributors punish only in selling or apply the punishment at both stages: buying and selling. Since there is no new wholesale price renegotiation, the punishment in the upstream market cannot be applied which makes both types of punishment strategies completely equivalent.

The second result of the proposition tells us that in the case of maximum upstream price rigidity, collusion at both stages is easier to sustain than collusion only at selling. This is a very strong result, because even in the most unfavorable scenario for punishing the deviator, when wholesale price cannot move up after a deviation, the upstream collaboration among retailers still renders collusion downstream more feasible. As we have proved, this result is valid for any degree of price differentiation between retailers.
4. Conclusions

Using a simple model of two retailers competing downstream that buy a homogeneous input from the same manufacturer, we found that cooperation at the purchase stage makes it easier to collude at selling downstream.

The implications of this result are relevant, since by acting cooperatively at purchasing an input, firms are facilitating themselves to collude at the retail level. So, the supposedly harmless joint purchasing agreement may hide an undesirable effect on welfare, which is inducing colluding downstream. Note that ceteris paribus, colluding at purchasing reduces wholesale prices, and part of it is passed through to consumers in the form of lower retail prices. However, when retailers play a repeated game, the stability of a collusive downstream plot is fostered by the existence of an upstream agreement. When retailers act jointly in negotiations with the producers, they obtain lower wholesale prices. This benefit, may be lost if one firm deviates and the punishment breaks also the joint negotiation with the producer. We further show that the above result is robust to scenarios of upstream price rigidity, where upstream price renegotiation cannot take place immediately after a firm has deviated from the collusive agreement.
Appendix

**Proof of Lemma 1:**

Part (i). Taking the partial derivatives of wholesale and retail prices with respect to differentiation parameter $\beta$:

$$\frac{\partial w^N}{\partial \beta} = \frac{-2\beta(4 + 2\beta - 5\beta^2 + \beta^3 + \beta^4)}{[(4 - \beta^2)(2 - \beta) - \beta^4]^2} \leq 0$$

$$\frac{\partial p^N}{\partial \beta} = -\frac{(1 - w^N) - (2 - \beta) \frac{\partial w^N}{\partial \beta}}{(2 - \beta)^2} = -\frac{(1 - w^N) + (2 - \beta) \frac{\partial w^N}{\partial \beta}}{(2 - \beta)^2} < 0$$

Given that $w^N(0) = 0.25$ and thus $1 - w^N > 0$

Part (ii). Taking the limit of the wholesale and retail prices when $\beta \to 1$, we obtain:

$$\lim_{\beta \to 1} w^N = 0$$

$$\lim_{\beta \to 1} p^N = \frac{0 + \lim_{\beta \to 1} w^N}{1} = 0$$

**Proof of Lemma 2:**

Part (i). Taking the partial derivatives of wholesale and retail prices with respect to differentiation parameter $\beta$, we get:

$$\frac{\partial w^B}{\partial \beta} = \frac{-(2 + \beta^2)}{(6 - \beta - 3\beta^2)^2} \leq 0$$

$$\frac{\partial p^B}{\partial \beta} = -\frac{(1 - w^B) - (2 - \beta) \frac{\partial w^B}{\partial \beta}}{(2 - \beta)^2} = -\frac{(1 - w^B) + (2 - \beta) \frac{\partial w^B}{\partial \beta}}{(2 - \beta)^2} < 0$$

Given that $w^B(0) = \frac{1}{6}$ and thus $1 - w^B > 0$

Part (ii). Taking the limit of the wholesale and retail prices when $\beta \to 1$, we obtain:

$$\lim_{\beta \to 1} w^B = 0$$

Drawn straightforward from equation (10).
Proof of Lemma 3:

Part (i). Taking the partial derivatives of wholesale and retail prices with respect to differentiation parameter $\beta$, we obtain:

$$ \frac{\partial w^s}{\partial \beta} = \frac{1}{(4-\beta)^2} > 0 \quad \land \quad \frac{\partial p^s}{\partial \beta} = \frac{1}{2} \frac{\partial w^s}{\partial \beta} > 0 $$

Part (ii). Taking the limit of the wholesale and retail prices when $\beta \to 1$, we obtain:

$$ \lim_{\beta \to 1} w^s = \frac{1}{3} $$

Drawn straightforward from equation (11), we get:

$$ \lim_{\beta \to 1} p^s = \frac{1 + \lim_{\beta \to 1} w^s}{2} = \frac{2}{3} $$

Proof of Lemma 4:

Part (i). Taking the partial derivatives of wholesale and retail prices with respect to differentiation parameter $\beta$, we get:

$$ \frac{\partial w^{BS}}{\partial \beta} = \frac{-1}{2(3-2\beta)^2} < 0 \quad \land \quad \frac{\partial p^{BS}}{\partial \beta} = \frac{1}{2} \frac{\partial w^{BS}}{\partial \beta} < 0 $$

(ii) Part (ii). Taking the limit of the wholesale and retail prices when $\beta \to 1$, we obtain:

$$ \lim_{\beta \to 1} w^{BS} = 0 $$

Obtained straightforward from equation (14).

$$ \lim_{\beta \to 1} p^{BS} = \frac{1 + \lim_{\beta \to 1} w^{BS}}{2} = \frac{1}{2} $$

Proof of Proposition 1:

We first demonstrate $w^s \geq w^N \geq w^B \geq w^{BS}$:

- $w^s \geq w^N \iff (4 - \beta^2)(2 - \beta) - \beta^4 \geq (2 + \beta)(1 - \beta)(4 - \beta)$
  $$ \iff 2 + \beta - \beta^3 \geq 0 $$
  $$ \iff 0 \leq \beta < 1 $$

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Now we demonstrate \( p^S \geq p^N \geq p^B \)

- \( p^S \geq p^N \iff (2 - \beta)(1 + w^S) \geq 2(1 - \beta + w^N) \)
  \[ \iff \beta(1 - w^S) + 2(w^S - w^N) \geq 0 \]
  \[ \iff w^S < 1 \land w^S \geq w^N \]

- \( p^N \geq p^B \iff (1 - \beta + w^N) \geq (1 - \beta + w^B) \)
  \[ \iff w^N \geq w^B \]

Additionally,
- \( p^S \geq p^{BS} \iff (1 + w^S) \geq (1 + w^{BS}) \)
  \[ \iff w^S \geq w^{BS} \]

- \( p^{BS} \geq p^B \iff (2 - \beta)(1 + w^{BS}) \geq 2(1 - \beta + w^B) \)
  \[ \iff \beta(1 - w^{BS}) + 2(w^{BS} - w^B) \geq 0 \]
  \[ \iff w^{BS} < 1 \land w^{BS} \geq w^B \]

For getting the relation between \( p^N \) and \( p^{BS} \) computing is needed. A computational equation solver helps us find a unique value for \( \beta \) such that: \( p^N - p^{BS} = 0 \). Then: \( p^{BS} > p^N \) if \( \beta > 0.22172 \) (See Graph 1).

**Proof of Proposition 2:**

Firstly, (i) and (ii) implies that \( \pi^N \leq \pi^B \leq \pi^{BS} \):

- \( \pi^{BS} \geq \pi^B \iff (2 - \beta)^2(1 - w^{BS})^2 \geq 4(1 - \beta)(1 - w^B)^2 \)
  \[ \iff 4(1 - \beta)[(1 - w^{BS})^2 - (1 - w^B)^2] + \beta^2(1 - w^{BS})^2 \geq 0 \]
  \[ \iff w^{BS} \leq w^B \]

- \( \pi^B \geq \pi^N \iff w^B \leq w^N \)

For getting the relation between \( \pi^N \) and \( \pi^S \) computing is needed. A computational equation solver helps us find a unique value for \( \beta \) such that: \( \pi^N - \pi^S = 0 \) in \( \beta = 0.65092 \), so \( \pi^S > \pi^N \) if \( \beta > 0.65092 \) (see Graph 2).
5. References


