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ENDOGENOUS DIFFERENTIAL INFORMATION IN FINANCIAL MARKETS

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ENDOGENOUS DIFFERENTIAL INFORMATION IN FINANCIAL MARKETS

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Abstract. We develop a two period general equilibrium model with incomplete financial markets and differential information. Making endogenous the traditional informational restriction on consumption, we allow agents to obtain information from physical and financial markets. Thus, the investment in financial promises and the trade of commodities in spot markets appear as natural channels to improve the information that an agent has about the realization of future states of nature.

Keywords. Incomplete Markets, Differential information, Enlightening equilibrium.
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1. Introduction

The modern theory of incomplete financial markets begins in seminal papers of Radner (1972), Drèze (1974) and Hart (1975), which extended the classical complete markets model of Arrow and Debreu (1954) introducing an incomplete set of financial promises that not necessarily allow for perfect risk sharing. This theory has been matter of research since then, and extensions to different scenarios were studied (see for instance, Geanakoplos (1990) and Magill and Quinzii (2008) for surveys of major results in this theory).

Particularly, the role of financial markets to communicate information for asymmetrically informed traders was studied, among others, by Polemarchakis and Siconolfi (1993), Rahi (1995) and Cornet and De Boisdeffre (2002). In Polemarchakis and Siconolfi (1993) the focus is given to show the existence of non-informative rational expectations equilibrium in nominal asset markets. On the other hand, equilibrium with enlightening prices was studied in two period economies by Rahi (1995), who models private information as signals about state of nature that will be reached, and shows that any structure of information compatible with non-arbitrage may be embedded in a rational expectations equilibrium. In a similar context, Cornet and De Boisdeffre (2002) assume that agents anticipate asset prices and, before the trade of commodities and assets, make a refinement of their signals by precluding arbitrage opportunities. Thus, a vector of asset prices is implementable as equilibrium only if the pooling information, which is obtained after the exclusion of arbitrage

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opportunities, is non-empty (see also De Boisdeffre (2007)). In particular, there are financial markets where only asset prices that fully reveal information are equilibria. However, when default is allowed and assets are collateralized, as in Geanakoplos (1997) or Geanakoplos and Zame (2002), it is not necessary that agents update information to assure equilibrium existence, since the obligation of borrowers to constitute collateral induce natural bounds on the amount of resources that an agent may obtain from arbitrage (as was proved by Petrassi and Torres-Martínez (2007)).

Different to contributions above, in this paper we model asymmetric information as a lack of knowledge about the state of nature that was realized, referred in the literature as differential information. Thus, an agent does not receive a signal that allows him to concentrate his contingent decisions in a subset of states of nature. However, after the realization of the uncertainty, the agent may not have information to distinguish the state of nature that was effectively reached.

Contrary to the traditional model of differential information of Radner (1968), in our model agents demand commodities in spot markets and, therefore, they may infer information about the state of nature that was realized at the second period when observe either assets payments or spot commodity prices. In fact, in the original model of Radner (1968) all negotiations are made at the same period and a complete set of future contingent contracts are available for trade. Thus, even when prices of future contracts on some commodities differ among states of nature, agents do not gain information about the state of nature that will be realized.[1]

Recently, in a model that incorporates differential information into the incomplete financial markets framework, Faias and Moreno-García (2010) do not take into account the effect of markets signals on individuals information. Essentially, they center the analysis in particular types of assets that do not reveal information and, also, in equilibrium prices that are compatible with the common *ex-ante* information. Therefore, individuals do not receive new information from physical or financial markets. With this focus on equilibria that only considers non-enlightening prices, they shown that the degree of real indeterminacy of equilibrium decreases in nominal asset markets. However, there is no reason to believe that markets will concentrate in this type of non-informative equilibrium allocations.

Consequently, we consider the information that agents may receive from physical and financial markets, through the negotiation of commodities in spot markets or the investment in financial promises. These activities may allow agents to improve their information about the state of nature that effectively occurs, using prices or asset deliveries as market signals. For this reason, we do not impose exogenous restrictions on consumption to make physical allocations compatible with the

[1]Along the paper we follow the interpretation of differential information given by Daher, Martins-da-Rocha and Vailakis (2007): today, each agent has a complete information structure about the set of possible states of nature, but he is not necessarily able to discern, tomorrow, which states of nature was realized.

information that agents have before the realization of the states of nature. Thus, in our model, we make individual's differential information endogenous.

More precisely, we assume that financial assets are numeraire and free of default. Also, we incorporate restrictions in the pattern of promises that every agent can do. Specifically, agents can not make promises that, to be honored, require information that they do not have at the moment in which the debt contract is signed. There are endogenous restrictions on consumption bundles, allowing agents to demand consumptions plans that are compatible with the final information, which include those obtained using markets signals. Since the restriction over consumption bundles implies that budget set correspondences do not have closed graph, to prove equilibrium existence, we internalize this restriction by require preferences to be represented by a separable utility function.

The rest of the paper is organized as follows: In Section 2 we present our model. Section 3 is devoted to discuss our main result and its Assumptions. In Section 4 we provide examples and conclude with a final section of remarks. The proof of our main results is relegated to an Appendix.

2. Model

Consider a two period economy where there is no uncertainty at the first period, $t = 0$, and one state of a finite set S is realized at the second period, $t = 1$. To shorten notations, let $S^* = \{0\} \cup S$ be the set of states of nature in the economy, identifying $s = 0$ as the only state of nature at the first period.

There is a finite set L of commodities that may be traded at each period in spot markets. Let $p_s = (p_{s,l}; l \in L)$ be the unitary commodity price at state of nature $s \in S^*$ and $p = (p_s; s \in S^*)$ the plan of commodity prices in the economy. We fix along the paper a bundle $\zeta = (\zeta_l; l \in L) \in \mathbb{R}^L_{++}$ and normalize its unitary price, at any state of nature, to be equal to one, $p_s \cdot \zeta = 1, \forall s \in S^*$. Thus, a plan of commodity prices will belong to $\mathcal{P} := \{(p_s; s \in S^*) \in \mathbb{R}^{L\times S^*}_+ : p_s \cdot \zeta = 1, \forall s \in S^*\}$.

Let J be a finite set of numeraire assets which are issued at $t = 0$. Each asset $j \in J$ has a unitary price q_j at the first period and make promises contingent to the states of nature in S, $R_j = (R_{s,j}; s \in S) \in \mathbb{R}^S_+$, which are measure in units of the bundle ζ. There also exists a riskless asset which is negotiated by a unitary price $\pi > 0$ at the first period and delivers the bundle ζ at every state of nature in the second period. We assume that there are no redundant assets, i.e., that the family of vectors $\{(1, \dots, 1)\} \cup \{R_j; j \in J\}$ is linearly independent. For convenience of notations, let (q, π) be the vector of asset prices in the economy, where $q := (q_j; j \in J) \in \mathbb{R}^J_+$.

There is a finite set of agents, denoted by I, that demand commodities and trade financial assets. Each agent $i \in I$ has preferences over consumption represented by a utility function $U^i : \mathbb{R}^{L\times S^*}_+ \to \mathbb{R}$ and receive, contingent to the period and the state of nature, initial endowments of commodities, which are given by $w^i = (w^i_s; s \in S^*) \in \mathbb{R}^{L\times S^*}_+$.

Agents may have incomplete information about the realization of the states of nature. We assume that the initial information that an agent $i \in I$ have about the realization of future states of nature is given by a partition of the set S, denoted by $\mathbb{P}^i$. Essentially, the agent i knows the set S of states of nature that may be reached at $t = 1$, but after the realization of the uncertainty, if state of nature s is realized, the agent i will distinguish between it and the state of nature s' if and only if s and s' belongs on different elements of $\mathbb{P}^i$. However, each agent may obtain additional information from commodity prices and asset payments and, therefore, their consumption only need to be compatible with their final information constraint. For this reason, given a partition $\mathbb{P}$ of S, we say that a vector $(v_s; s \in S) \in \mathbb{R}^{L \times S}$ as $\mathbb{P}$-measurable if $v_s = v_{s'}$ for any pair of state of nature s and s' that belongs on a same element of the partition $\mathbb{P}$.

We suppose that the vector $w^i_{-0} := (w^i_s; s \in S)$ is $\mathbb{P}^i$-measurable. That is, all the information that an agent i may obtain from his endowments variability is contained in $\mathbb{P}^i$.

As we said above, the final information that an agent has about the realization of states of nature will be endogenous. More precisely, in our framework agents obtain information from commodity prices and asset payments. Thus, at the first period, if *ex-ante* an agent $i \in I$ does not distinguish between the states of nature s and s', he may distinguish *ex-post* them if either (i) commodity prices p_s and $p_{s'}$ are different, or (ii) the agent buys at $t = 0$ an asset that has different unitary payments at states of nature s and s'. Thus, as in the real world, in our model commodity and asset prices are a natural market signals that reveals information to traders.

We assume that actions associated to a financial contract need to be compatible with the information that is available at the moment in which these actions are taken, not with the information that may be obtained after the realization of these actions. Precisely, buyers of an asset pay today and expect to receive payments tomorrow, thus their do not execute any action at period $t = 1$. For these reason, there is no restriction over investment opportunities. On the other hand, a seller of an asset promises to make contingent payments at $t = 1$. Therefore, at the moment where a payment is due, the seller need to known the state of nature that was reached, independent of the signals that commodity prices may give. Consequently, we restrict agents using informational dependent credit constraints. In other case, an investor that believes that he will obtain information about the realization of an state of nature through asset payments, may not receive any financial return because borrowers are also waiting to obtain information about the state of nature to honor the promise.

More precisely, given a plan of prices $p \in \mathcal{P}$, an agent $i \in I$ can only choose short-positions on assets that belongs to $J(\mathbb{P}^i) := \{j \in J : (R_{s,j}; s \in S) \, \text{is} \, \mathbb{P}^i\text{-measurable}\}$. The set $J(\mathbb{P}^i)$ only takes into account assets in J, because independently of their private information agents knows the necessary information to honor a debt in the risk-free asset.

Agents takes commodity and assets prices as given, $((p_s; s \in S^*), \pi, q) \in \mathcal{P} \times \mathbb{R}_+ \times \mathbb{R}^J_+$. Each agent $i \in I$ makes decisions about consumption, choosing contingent bundles $(x^i_s; s \in S^*) \in \mathbb{R}^{L\times S^*}_+$. Also, he makes financial positions $(z^i, \theta^i, \varphi^i) \in \mathbb{R} \times \mathbb{R}^J_+ \times \mathbb{R}^J_+$, where z^i is the quantity of the riskless asset that he buys or sells at $t = 0$. Analogously, $(\theta^i, \varphi^i) = ((\theta^i_j, \varphi^i_j); j \in J)$ are the quantities of assets that agent $i \in I$ buys and sells. Associated to these financial positions, at any state of nature in the second period, agent $i \in I$ receives the returns of his positions or pay his promises, $z^i + \sum_{j\in J} R_{s,j}(\theta^i_j - \varphi^i_j)$.

Note that, given commodity prices for the states of nature in the second period, $p_{-0} := (p_s; s \in S)$, when an agent $i \in I$ decides to buy quantities θ^i_j of each asset $j \in J$, he will distinguish two states of nature s and s' at the second period if and only if at least one of the following three conditions hold:

(a) Both states of nature were in different elements of $\mathbb{P}^i$, i.e., were distinguishable *ex-ante*.

(b) There is $j \in J$, such that $R_{s,j}\theta^i_j \neq R_{s',j}\theta^i_j$. That is, financial markets allow the agent to distinguish these states when he receive the payments of some of the negotiated assets.

(c) There exists $l \in L$ such that $p_{s,l} \neq p_{s',l}$. That is, physical markets allow the agent to distinguish these states of nature when there is at least one commodity for which unitary prices are different between these states.

Thus, the final information of agent i when he choose a portfolio θ^i is given by a partition $\mathbb{P}^i(p_{-0}, \theta^i)$ as finer as $\mathbb{P}^i$ in which two states of nature are indistinguishable if and only if the three conditions above do not hold. As is usual in the differential information literature, in our model agents restrict themselves to consume bundles that are compatible with the (final) available information, $\mathbb{P}^i(p_{-0}, \theta^i)$. Essentially, this occurs since any agent knows that if some of two states of nature $\{s, s'\}$ that he can not distinguish occurs, he will not have signals to choose the bundle of consumption when $x^i_s \neq x^i_{s'}$.

By requiring $(x^i_s; s \in S)$ to be $\mathbb{P}^i(p_{-0}, \theta^i)$-measurable we capture the effect that the investment in financial assets has in the consumption possibilities of agent i. Thus, a reason to invest in some asset may be the interest of the agent to increase the set of bundles which are both budgetary and informationally implementable.

Let $\Gamma(\mathbb{P}^i)$ be the set of market-admissible plans for agent $i \in I$. That is, the set of vectors $\left(x^i, z^i, \theta^i, \varphi^i\right) \in \mathbb{R}^{L\times S^*}_+ \times \mathbb{R} \times \mathbb{R}^J_+ \times \mathbb{R}^J_+$ such that $\varphi_j = 0$ for any $j \notin J(\mathbb{P}^i)$. Note that Γ^i is non-empty, closed and convex.

Given prices $\left((p_s; s \in S^*), q, \pi\right) \in \mathcal{P} \times \mathbb{R}^J_+ \times \mathbb{R}_+$, the objective of any agent $i \in I$ is to maximize his utility function by choosing an allocation in his budget set, which is denoted by $B^i(p, q, \pi)$ and is defined as the collection of vectors $(x^i, z^i, \theta^i, \varphi^i) \in \Gamma(\mathbb{P}^i)$ such that both $(x^i_s; s \in S) \in \mathbb{P}^i(p_{-0}, \theta^i)$

and

$$\begin{aligned} p_0 x_0^i + \pi z^i + \sum_{j\in J} q_j \theta_j^i &\leq p_0 w_0^i + \sum_{j\in J} q_j \varphi_j^i, \\ p_s x_s^i + \sum_{j\in J} R_{s,j}\varphi_j^i &\leq p_s w_s^i + z^i + \sum_{j\in J} R_{s,j}\theta_j^i, \ \forall s \in S. \end{aligned}$$

Definition 1. *An equilibrium for the economy with endogenous differential information is given by a vector of unitary prices* $(\overline{p},\overline{q},\overline{\pi}) \in \mathcal{P}\times\mathbb{R}_+^J\times\mathbb{R}_+$ *jointly with individual allocations* $\left(\overline{x}^i,\overline{z}^i,\overline{\theta}^i,\overline{\varphi}^i\right)_{i\in I} \in \prod_{i\in I}\Gamma(\mathbb{P}^i)$ *such that,*

(i) Every agent $i \in I$*, the vector* $\left(\overline{x}^i,\overline{z}^i,\overline{\theta}^i,\overline{\varphi}^i\right)$ *maximizes the utility function* $U^i : \mathbb{R}_+^{L\times S^*} \to \mathbb{R}_+$ *among the allocations on his budget set* $B^i(\overline{p},\overline{q},\overline{\pi})$*.*

(ii) The following markets clearing conditions hold,

$$\sum_{i\in I}\left(\overline{x}_s^i - w_s^i\right) = 0, \quad \forall s \in S^*; \qquad \sum_{i\in I}\overline{z}^i = 0, \qquad \sum_{i\in I}(\overline{\theta}_j^i - \overline{\varphi}_j^i) = 0, \ \forall j \in J.$$

3. Existence of equilibrium

Our main result about existence of equilibrium is,

Theorem 1. *Suppose that the following assumptions hold,*

(A1) *For each agent* $i \in I$*, the utility function* $U^i : \mathbb{R}_+^{L\times S^*} \to \mathbb{R}$ *is continuous, strictly concave and strictly increasing.*

(A2) *For each agent* $i \in I$*,* $w^i \in \mathbb{R}_{++}^{L\times S^*}$*.*

(A3) *The utility function of any agent* $i \in I$ *satisfies the following asymptotic property,*

$$\lim_{r\to+\infty} U^i(x_0 + r\cdot\zeta,(x_s;s\in S)) = +\infty, \quad \forall (x_s; s\in S^*) \in \mathbb{R}_{++}^{L\times S^*}.$$

(A4) *For each agent* $i \in I$*,* $U^i(x_s; s\in S^*) := \sum_{s\in S}\alpha_s u^i(x_0,x_s)$*, where* $u^i : \mathbb{R}_+^L \times \mathbb{R}_+^L \to \mathbb{R}_+$*, and the parameter* $\alpha_s > 0$ *is the probability that agent* i *gives to state of nature* $s \in S$ *occurs.*

Then, there exists an equilibrium.

Depending on initial information, agents may not have access to some credit markets. Thus, if we normalize commodity and asset prices in such form that their are in the simplex, then budget set correspondences may have empty interior. Thus, to prove equilibrium existence, we normalize commodity prices to belongs to $\mathcal{P}$ and, therefore, we need to determine upper bounds on asset prices. For these reason, we include Assumption (A3), since it allows us to find endogenous bounds in the price of the risk-free asset, in order to also bound asset prices from above using non-arbitrage conditions. Moreover, conditions (A2) and (A3) allow us to prove lower hemicontinuity of budget set correspondences, because assure that interior budget set correspondences have non-empty values.

Note that, the restriction of information over consumption plans implies that budget set correspondence of any agent do not have closed graph, an important property that is needed to prove equilibrium existence using the traditional generalized game approach.[2] Thus, we need to assure the compatibility between the consumption plan and the final information without impose explicitly in the budget set correspondences of agents this informational restriction. For these reasons, we concentrate our analysis in preferences that satisfy Assumption (A4), because with this hypothesis an *informational unrestricted* agent i consumes different bundles of commodities at two states $(s, s') \in S \times S$ only if these states of nature are distinguishable in $\mathbb{P}^i(\overline{p}_{-0}, \overline{\theta}^i)$.

In fact, if s and s' are in the same element of $\mathbb{P}^i(\overline{p}_{-0}, \overline{\theta}^i)$, then (by definition) both states of nature are also in the same element of $\mathbb{P}^i$, $\overline{p}_s = \overline{p}_{s'}$, and $R_{s,j}\overline{\theta}^i_j = R_{s',j}\overline{\theta}^i_j$, $\forall j \in J$. Thus, given the equilibrium bundles $\overline{x}^i_s$ and $\overline{x}^i_{s'}$, for any $\lambda \in [0, 1]$ the vector $x(\lambda) := \lambda\overline{x}^i_s + (1 - \lambda)\overline{x}^i_{s'}$ is budget feasible at both states of nature. By Assumption (A4) and the non-distinguishability of states of nature s and s', it follows that, $u^i(\overline{x}^i_0, \overline{x}^i_s) = u^i(\overline{x}^i_0, \overline{x}^i_{s'})$. Thus, suppose that $\overline{x}^i_s \neq \overline{x}^i_{s'}$. If agent i changes his equilibrium bundles at s and s' by $x(\lambda)$, $\lambda \in (0, 1)$, then he improves his utility level (a consequence of the strictly concavity of his utility function). A contradiction.

Moreover, since the separability of utility functions imposed in Assumption (A4) endogenize the information compatibility requirement, we do not need to assume that, for any state of nature $s \in S$ there is at least one agent $i \in I$ that distinguish it, $\{s\} \in \mathbb{P}^i$. A traditional assumption on static general equilibrium models with differential information, used to assure that (under monotonicity of preferences) the equilibrium price of any contingent commodity contract is strictly positive.

4. Some examples

In our model, we may have equilibria in which agents gain information from financial markets because, contrary to Faias and Moreno-García (2010, Assumption (A.4)), we do not impose any assumption about measurability of asset payments. Also, we allow for equilibria with enlightening commodity prices, since there is no reason to concentrate in a refinement concept of equilibria where the plan of commodity prices is measurable with respect to the common information partition.

On the other hand, our debt constraints, which are dependent of agents initial information, are imposed in order to assure that borrowers have sufficient information to known, after the realization

[2] Indeed, fix an agent $i \in I$ that is not fully informed (i.e., $\mathbb{P}^i \neq \{\{s\}; s \in S\}$), and consider a sequence of prices $(p_n)_{n\geq 1} \subset \mathcal{P}$ that converges to a price $p \in \mathcal{P}$. Also, suppose that, for each $n \geq 1$, p_n is $\mathbb{Q}$-measurable, with $\mathbb{Q}$ strictly finer than $\mathbb{P}^i$, and p is $\mathbb{P}^i$-measurable. If we suppose that $z^i = \theta^i = \varphi^i = 0$ and $x^i = (w^i_0, (\alpha_s w^i_s; s \in S))$, where the vector $(\alpha_s; s \in S) \in (0, 1)^S$ is $\mathbb{P}^i(p_n, 0) = \mathbb{Q}$ measurable, the bundle $(x^i, z^i, \theta^i, \varphi^i)$ belongs to $B^i(p_n, q, \pi)$, for any $n \geq 1$ and independent of the vector $(q, \pi) \in \mathbb{R}^J_+ \times \mathbb{R}_+$. However, $(x^i, z^i, \theta^i, \varphi^i) \notin B^i(p, q, \pi)$.

of the uncertainty, if there is some debt to be honored. In other cases, may appear cycles on the decision process of payments delivery, promoted by the absence of information about the state of nature that was realized.

These possibilities are illustrated in the following examples.

Example 1. Consider an economy that satisfies Assumptions (A1)-(A4), with two commodities, no financial markets and utility functions given by

$$U^i((x_s; s \in S^*)) = x_{0,1}^{\beta} x_{0,2}^{1-\beta} + \sum_{s \in S} \alpha_s x_{s,1}^{\beta} x_{s,2}^{1-\beta}, \quad \forall i \in I,$$

where $\beta \in (0,1)$. The first-order conditions of consumer's problem at state $s \in S$ would imply that, at any equilibrium price $\overline{p}_s$,

$$\frac{\overline{p}_{s,1}}{\overline{p}_{s,2}} = \frac{\beta}{1-\beta} \frac{W_{s,2}}{W_{s,1}},$$

where $W_{s,l} = \sum_{i \in I} w_{s,l}^i$.

Suppose that there is an uninformed agent $i_0 \in I$ (i.e. $\mathbb{P}^{i_0} = \{S\}$). Then, equilibrium prices will be non-informative, in the sense that are measurable with respect to the common information partition, if and only if the relative degree of commodity scarcity is constant at the second period, $\frac{W_{s,2}}{W_{s,1}} = \frac{W_{s',2}}{W_{s',1}}$, $\forall (s,s') \in S \times S$, which is a restrictive hypothesis. Moreover, for any economy in which this condition does not hold, any equilibrium price will reveal information (at least for the uninformed agent i_0).

Note that, if we allow for financial markets in this economy, to assure that asset payments do not reveal information, we need to restrict it to be measurable with respect to the common information. Thus, only risk-free assets are available to be traded. A strong assumption, specially when the number of states of nature is high. In other words, any condition about measurability of asset payments with respect to the common information partition of the economy, strongly increase the degree of incompleteness of financial markets. Thus, when markets have a good level of financial innovation, it is not credible that asset payments do not reveal information.

Example 2. In this example we illustrate the importance of the existence in our model of informational dependent debt constraints. For simplicity, consider an economy with only one commodity, three states of nature at $t = 1$, denoted by $\{u, m, d\}$, and two agents that only receive utility for consumption at the second period. Also, they do not have any initial endowment at $t = 0$. Thus, utility functions and endowments are given by

$$U^1(x_u, x_m, x_d) = \sqrt{x_u} + \sqrt{\frac{x_m}{3}} + 2\sqrt{x_d}, \qquad (w_u^1, w_m^1, w_d^1) = (0, 2, 2);$$

$$U^2(x_u,x_m,x_d)=\sqrt{x_u}+\sqrt{x_m}+2\sqrt{\frac{x_d}{3}}, \qquad (w_u^2,w_m^2,w_d^2)=(2,2,2).$$

There are two numeraire assets in the economy. One of them has a unitary price q_1 at the first period and promise to deliver one unit of the commodity at states of nature $\{u,m\}$. The other asset is an Arrow security contingent to state of nature $s=d$, which is negotiated at $t=0$ for a unitary price q_2. Therefore, if there is no debt constraints in the economy, the first period budget constraint of agent $i\in\{1,2\}$ is given by $q_1z_1^i+q_2z_2^i=0$, where z_j^i denotes the position of agent i on asset $j\in\{1,2\}$. Note that, in the absence of debt constraints it is not necessary to identify long and short position by different notations.

Assume that unitary prices are given by $\overline{q}_1=\overline{q}_2=1$. Then, the allocations

$$(z_1^1,z_2^1,x_u^1,x_m^1,x_d^1)=(1,-1,1,3,1), \qquad (z_1^2,z_2^2,x_u^2,x_m^2,x_d^2)=(-1,1,1,1,3).$$

constitute an equilibrium for the economy.

We argue that, depending on agents initial informations, may be not credible that this equilibrium allocation could be implemented. In fact, if we assume that $\mathbb{P}^1=\mathbb{P}^2=\{\{u\},\{m,d\}\}$, then to pay his debt, agent $i=1$ needs to observe that the state of nature $s=d$ was realized. To do this, he needs to believe that the payment associate to his long position will be always honored. Analogously, to pay his debt at states of nature $s\in\{u,m\}$, agent $i=2$ needs to believe that asset $j=2$ do not gives default. Thus, a cycle of interdependent decisions appears, that blocks any payment to be made, given the partial information that borrowers have.

Thinking in these kind of situations, we impose our informational dependent debt constraints, namely $z_j^i\geq 0$ if $j\notin J(\mathbb{P}^i)$, for each agent $i\in\{1,2\}$. In this context, equilibrium financial debt and physical endowments are compatible with our model if and only if agent $i=1$ is fully informed about the realization of states of nature and agent $i=2$ is either fully informed too or has a partition of information given by $\mathbb{P}^2=\{\{u,m\},\{d\}\}$. In other cases, at least one of the agents does not have fully access to credit markets and, therefore, in equilibrium agent will consume their initial endowments.

5. Concluding remarks

In this paper we extend the model of competitive markets with differential information introduced by Radner (1968), to allow for sequential trade in incomplete financial markets. Thus, agents buy commodities in spot markets and receive signals from commodity prices and asset payments, that allow them to improve the private information about the realization of states of nature. Also, initial restrictions on information induce natural debt constraints that avoid cycles that undetermine financial actions. Since the restrictions over consumption imposed by the information are made endogenous, we do not need to restrict the number of assets available for trade or its contingent payments.

There are some interesting issues that could be matter of future research. For instance, our model may be extended to more than two periods and the discussion of the effects that default has over (informational) debt constraints could be relevant. In fact, when agents live for more than two periods, the investment in financial promises may have effects not only over immediate future consumption but also over future credit opportunities, as informed traders have more access to credit markets. However, any attempt to include multiple periods in our framework comes with a model that describes how the information evolves through the time. On the other hand, if we allow for default and protect investors by either penalize agents that do not honor their promises or demand collateral guarantees from borrowers (as in Dubey, Geanakoplos and Shubik (2005) or Geanakoplos and Zame (2002)), the credit restrictions presented in this paper may be relaxed as a consequence of the existence of payment enforcement mechanism.

Appendix: Proof of Theorem 1

To prove the existence of an equilibrium in our economy, we first define a generalized game in which agents maximize utility functions in truncated budget sets and auctioneers choose prices in order to maximize the value of the excess of demand in commodity and financial markets.

We prove that this generalized game has a Cournot-Nash equilibrium. Also, when the upper bounds that truncate by above the allocations of the budget set are high enough, any equilibrium of the generalized game will be an equilibrium of our economy.

The generalized game $\mathcal{G}(n,Q,X,Z,\Theta,\Phi)$. Given any vector $(n,Q,X,Z,\Theta,\Phi)\in\mathbb{N}^6$, we define a game characterized by the following set of players and strategies.

Set of players. There is a finite set of players constituted by,

(i) The set of agents of the economy, I.

(ii) An auctioneer, $h(s)$, for each $s\in S^*$.

We denote the set of players by $H=I\cup H(S^*)$ where $H(S^*):=\{h(s):s\in S^*\}$.

Sets of strategies. Given $\overline{W}:=\max_{(s,l)\in S^*\times L}\sum_{i\in I}w^i_{s,l}$, define

$$K(X,Z,\Theta,\Phi)=\left\{(x_0,(x_s;s\in S),z,\theta,\varphi)\in[0,X]^L\times[0,2\overline{W}]^{S\times L}\times[-Z,Z]\times[0,\Theta]^J\times[0,\Phi]^J\right\},$$

and, for any $s\in S^*$, let $\mathcal{P}_s=\{p\in\mathbb{R}^L_+ : p\cdot\zeta=1\}$. The set of strategies for the players in the generalized game, $(\overline{\Gamma}^h;h\in H)$, are given by,

(i) For each $h\in I$, $\overline{\Gamma}^h=K(X,Z,\Theta,\Phi)\cap\Gamma(\mathbb{P}^h)$.

(ii) For $h = h(0)$, $\overline{\Gamma}^h = \mathcal{P}_0 \times [0, n] \times [0, Q]^{\#J}$

(iii) For $h = h(s)$, with $s \in S$, $\overline{\Gamma}^h = \mathcal{P}_s$.

For simplicity, let $\eta^h = \left(x^h, z^h, \theta^h, \varphi^h\right) \in \overline{\Gamma}^h$ be a generic vector of strategies for a player $h \in I$; (p_0, π, q) will denote a generic strategy for the player $h(0)$; and p_s a generic strategy for a player $h(s)$, with $s \in S$. Finally, let $\overline{\Gamma} = \prod_{h \in H} \overline{\Gamma}^h$ be the space of strategies of $\mathcal{G}\left(n, Q, X, Z, \Theta, \Phi\right)$. A generic element of $\overline{\Gamma}$ is denoted by (p, π, q, η), where $\eta := \left(\eta^h; h \in I\right)$ is a generic element of $\prod_{i \in I} \overline{\Gamma}^i$.

Admissible strategies. Strategies effectively chosen for the players depend on the actions taken by the other players, throughout a correspondence of admissible strategies $\phi^h : \overline{\Gamma}_{-h} \twoheadrightarrow \overline{\Gamma}^h$, where $\overline{\Gamma}_{-h} = \prod_{h' \neq h} \overline{\Gamma}^{h'}$. Let $(p, \pi, q, \eta)_{-h}$ be a generic element of $\overline{\Gamma}_{-h}$. We suppose that,

(i) If $h \in I$, $\phi^h \left[(p, \pi, q, \eta)_{-h}\right] = B^h(p, \pi, q) \cap \overline{\Gamma}^h$.

(ii) If $h \in H(S^*)$, $\phi^h \left[(p, \pi, q, \eta)_{-h}\right] = \overline{\Gamma}^h$.

Objective functions. Each player is characterized by an objective function $F^h : \overline{\Gamma}^h \times \overline{\Gamma}_{-h} \to \mathbb{R}_+$. We assume that,

(i) When $h \in I$ and $\eta^h = \left(x^h, z^h, \theta^h, \varphi^h\right) \in \overline{\Gamma}^h$, then $F^h\left(\eta^h; (p, \pi, q, \eta)_{-h}\right) = U^i\left(x^h\right)$.

(ii) If $h = h(0)$ and $(p, \pi, q) \in \overline{\Gamma}^h$, then

$$F^h\left((p_0, \pi, q); (p, \pi, q, \eta)_{-h}\right) := p_0 \sum_{i \in I} \left(x_0^i - w_0^i\right) + \pi \sum_{i \in I} z^i + \sum_{i \in I} \sum_{j \in J} q_j(\theta_j^i - \varphi_j^i).$$

(iii) If $h(s) \in H(S^*) \setminus \{h(0)\}$ and $p_s \in \overline{\Gamma}^h$, then $F^h\left(p_s; (p, \pi, q, \eta)_{-h}\right) := p_s \sum_{i \in I}\left(x_s^i - w_s^i\right)$.

We define the correspondence of optimal strategies for each $h \in H$, $\Psi^h : \overline{\Gamma}_{-h} \twoheadrightarrow \overline{\Gamma}^h$ as

$$\Psi^h\left((p, \pi, q, \eta)_{-h}\right) := \underset{y \in \phi^h\left((p, \pi, q, \eta)_{-h}\right)}{\arg\max} F^h\left(y; (p, \pi, q, \eta)_{-h}\right).$$

Finally, let $\Psi : \overline{\Gamma} \twoheadrightarrow \overline{\Gamma}$ be the correspondence of optimal responses of the game, which is given by $\Psi(p, \pi, q, \eta) = \prod_{h \in H} \Psi^h\left((p, \pi, q, \eta)_{-h}\right)$.

Definition 3. *A Cournot-Nash equilibrium for the generalized game $\mathcal{G}\left(n, Q, X, Z, \Theta, \Phi\right)$ is given by a strategy profile $(\overline{p}, \overline{\pi}, \overline{q}, \overline{\eta}) \in \Psi(\overline{p}, \overline{\pi}, \overline{q}, \overline{\eta}) \subset \overline{\Gamma}$.*

In order to prove the existence of equilibrium in the generalized game, we need some properties of the admissible strategies correspondences that the following lemma gives.

Lemma 1. *Under Assumption (A2), admissible strategies correspondences, $\left(\phi^h; h \in H\right)$, are non-empty and continuous. Moreover, these correspondences have compact and convex values.*

Proof. For each player $h \in H(S^*)$, the correspondence of admissible strategies is constant and, therefore, it is continuous and non-empty. Also, by definition, its values are compact and convex.

On the other hand, for each player $h \in I$, it follows from the definition of the budget set that the correspondence of admissible strategies ϕ^h has non-empty, compact and convex values. Since the graph of this correspondence is closed, we obtain upper hemicontinuity. To assure the lower hemicontinuity of ϕ^h, we consider the correspondence $\mathring{\phi}^h\left((p,\pi,q,\eta)_{-h}\right) := int_{K(X,Z,\Theta,\Phi)}\, B^h(p,\pi,q)$, which associates to a vector of commodity and asset prices the set of allocations in $K(X,Z,\Theta,\Phi)$ that satisfy all the budget restrictions of agent h as strict inequalities. Note that, by Assumption (A2), this correspondence has non-empty values and open graph. Therefore, it is lower hemicontinuous. We know that the closure of $\mathring{\phi}^h\left((p,\pi,q,\eta)_{-h}\right)$, which is equal to $\phi^h\left((p,\pi,q,\eta)_{-h}\right)$, is also lower hemicontinuous. Therefore, correspondences of admissible strategies $(\phi^h; h \in I)$ are continuous. □

Proposition 1. *Under Assumptions (A1) and (A2) the set of Cournot-Nash equilibria for the game $\mathcal{G}\left(n,Q,X,Z,\Theta,\Phi\right)$ is non-empty.*

Proof. By Assumption (A1), each objective function in the game is continuous in all variables and quasi-concave in its own strategy. Also, the sets of strategies are non-empty, compact and convex.[3] By Lemma 1, admissible correspondences are continuous with non-empty, convex and compact-values. Thus, we can apply Berge's Maximum Theorem to assure that, for each player $h \in H$ the correspondence of optimal strategies, Ψ^h, is upper-hemicontinuous with non-empty, convex and compact values. Thus, the correspondence Ψ has closed graph with non-empty, compact and convex values. Applying Kakutani's Fixed Point Theorem to Ψ we conclude the proof. □

We will prove that, for vectors (n,Q,X,Z,Θ,Φ) in which coordinates are high enough, any equilibrium of the generalized game is an equilibrium for our economy. However, we need to previously find endogenous upper bounds for equilibrium variables.

[3]Note that, the restrictions on borrowing to make financial debt compatible with individual initial information are included in the definition of the set $\Gamma(\mathbb{P}^h)$. This set is closed, convex and non-empty. When we intersect it with $K(X,Z,\Theta,\Phi)$, to obtain $\overline{\Gamma}^h$, we have also compacity.

Lemma 2. *For each $s \in S$, fix a vector $(p_s, w_s, x_s) \in \mathcal{P}_s \times \mathbb{R}^L_+ \times \mathbb{R}^L_+$, with $x_s < \overline{W}$. Then, there exists $A > 0$ such that, any allocations $(z, (\kappa_j; j \in J)) \in \mathbb{R} \times \mathbb{R}^J$ satisfying*

$$p_s x_s = p_s w_s + z + \sum_{j \in J} R_{s,j} \kappa_j, \ \forall s \in S;$$

is bounded by A, i.e., belongs on $[-A, A]^{\#J+1}$.

Furthermore, the bound A only depends on $((\overline{W}, w_s, R_{s,j}); (s,j) \in S \times J)$.

Proof. Note that, as S (respectively, J) is a finite set, we can abuse of the notation and identify it with $\{1, \ldots, S\}$ (respectively, $\{1, \ldots, J\}$). Thus, we can rewrite the conditions in the statement of the Lemma in a matricial form:

$$\begin{bmatrix} p_1(x_1 - w_1) \\ \vdots \\ p_S(x_S - w_S) \end{bmatrix} = \begin{bmatrix} R_{1,1} & \cdots & R_{1,J} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ R_{S,1} & \cdots & R_{S,J} & 1 \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_J \\ z \end{bmatrix}$$

Since there is no redundant assets in the economy, we have that $J + 1 \leq S$. Moreover, we can find a non-singular sub-matrix of dimension $(J+1) \times (J+1)$. Specifically, we may assume, without loss of generality, that this matrix is given by

$$B = \begin{vmatrix} R_{1,1} & \cdots & R_{1,h} & \cdots & R_{1,J} & 1 \\ \vdots & & \ddots & & \vdots & \vdots \\ R_{J+1,1} & \cdots & R_{J+1,h} & \cdots & R_{J+1,J} & 1 \end{vmatrix}$$

Thus, we have that

$$\begin{bmatrix} p_1(x_1 - w_1) \\ \vdots \\ p_{J+1}(x_{J+1} - w_{J+1}) \end{bmatrix} = B \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_J \\ z \end{bmatrix}$$

By Cramer Rule,

$$z = \frac{det(B(y, J+1))}{\det(B)}, \qquad \kappa_j = \frac{det(B(y,j))}{\det(B)}, \ \forall j \in \{1, \ldots, J\},$$

where $y = (p_1(x_1 - w_1), \ldots, p_{J+1}(x_{J+1} - w_{J+1}))$ and $B(y,j)$ is the matrix obtained by change, in the matrix B, the j-ith column for the vector y. Since (i) the determinant is a continuous function; (ii) the vector y depends continuously of $((p_s, x_s); s \in S)$; and (iii) vectors $((p_s, x_s, w_s); s \in S)$ are in a compact space, it follows that vector $(z, (k_j; j \in J))$ is bounded, independently of the value of $((p_s, x_s, w_s); s \in S)$. Thus, there exists $A > 0$ which satisfies the conditions of the lemma and depends on $((\overline{W}, w_s, R_{s,j}); (s,j) \in S \times J)$. $\square$

Following the notation of the previous lemma, define $(\overline{Z},\overline{\Theta},\overline{\Phi}) = 2A(1,1,1)$.

The next two lemmas are used to prove that equilibrium asset prices of the generalized game are uniformly bounded. For convenience of notations, let $W_0 = (W_{0,l}; l \in L)$ be the vector of aggregated physical resources at $t = 0$, where $W_{0,l} := \sum_{i\in I} w^i_{0,l}$.

Lemma 3. *Under Assumptions (A1)-(A3), given $(\overline{p},\overline{\pi},\overline{q}) \in \mathcal{P} \times \mathbb{R}_+ \times \mathbb{R}^J_+$, suppose that there exists an optimal solution $(\overline{x}^i,\overline{z}^i,\overline{\theta}^i,\overline{\varphi}^i) \in \Gamma^i$ for the individual problem of some agent $i \in I$ such that $\overline{x}^i_0 \leq W_0$ and $\overline{x}^i_{s,l} \leq 2\overline{W}$, $\forall (s,l) \in S \times L$. Then, there exists $\overline{n}$ such that $\overline{\pi} < \overline{n}$.*

Proof. Define $\varepsilon = \min_{(s,l,i)\in S\times L\times I}\ w^i_{s,l}$, which is strictly positive as a consequence of Assumption (A2). Suppose that an agent $i \in I$ borrows $\frac{\varepsilon}{2}$ units of the risk-free asset, which report for him resources in the first period equal to $\frac{\overline{\pi}\varepsilon}{2}$. Thus, he may consume at the first period the bundle $w^i_0 + \frac{\overline{\pi}\varepsilon}{2}\,\zeta$ (remember that $\overline{p}_0 \cdot \zeta = 1$). Therefore, we need to have

$$U^i\left(w^i_0 + \frac{\overline{\pi}\varepsilon}{2}\,\zeta, \left(w^i_{s,l} - \frac{\varepsilon}{2}\right)_{(s,l)\in S\times L}\right) \leq U^i(\overline{x}^i) < U^i\left(W_0, (2\overline{W}(1,\ldots,1))_{s\in S}\right).$$

It follows from Assumptions (A1), (A2) and (A3) that there exists $\overline{n}$ such that $\overline{\pi} < \overline{n}$. □

We define $\overline{X} = 2(1 + \overline{n}\sum_{l\in L}\zeta_l)\overline{W}$.

Note that, for any $X > \overline{X}$ and $n > \overline{n}$, in the associated generalized game $\mathcal{G}\,(n,Q,X,Z,\Theta,\Phi)$ any player $h \in I$ may demand at the first period the bundle used in the proof of Lemma 3. Thus, in this type of generalized game, the existence of an optimal plan satisfying the conditions of lemma above will imply that the unitary price of the risk-free asset is bounded from above by $\overline{n}$.

Lemma 4. *There exists $\overline{Q} > 0$ such that, in any equilibrium of the game $\mathcal{G}\,(n,Q,X,Z,\Theta,\Phi)$ with $(n,Q,X,Z,\Theta,\Phi) \gg (\overline{n},\overline{Q},\overline{X},\overline{Z},\overline{\Theta},\overline{\Phi})$, if there is some agent $i \in I$ such that (i) $\overline{x}^i_0 \leq W_0$; (ii) $\overline{x}^i_{s,l} \leq 2\overline{W}$, $\forall (s,l) \in S \times L$; and (iii) for some $j \in J$, $(\overline{\theta}^i_j,\overline{\varphi}^i_j) \in \mathbb{R}_{++} \times \{0\}$; then the unitary price $\overline{q}_j$ is bounded from above by $\overline{Q}$.*

Proof. Since $\overline{\theta}^i_j > 0$, applying Kuhn-Tucker Theorem to the optimization problem of agent i, we have that, $\overline{q}_j = \sum_{s\in S}\frac{\gamma^i_s}{\gamma^i_0}R_{s,j}$, where $\left(\gamma^i_s; s \in S\right)$ is the vector of Lagrange multipliers of the agent i which are associated to the budget constraints. Note that, it follows from Lemma 2 that $\overline{\theta}^i_j < \overline{\Theta} < \Theta$. For this reason, we do not include–in the first order condition above–the shadow price associated to the upper bound constraint on the long-position of $j \in J$. Moreover, since there is no restrictions on sales on risk-free asset, $\overline{\pi} = \sum_{s\in S}\frac{\gamma^i_s}{\gamma^i_0}$.

Therefore,

$$\overline{q}_j = \sum_{s\in S} \frac{\gamma_s^i}{\gamma_0^i} R_{s,j} \leq \overline{\pi} \max_{(s,j')\in S\times J} R_{s,j'} < \overline{Q} := \overline{n} \max_{(s,j')\in S\times J} R_{s,j'},$$

where $\overline{n}$ is the upper bound for the unitary price of risk-free asset, which was found in the previous lemma. □

Note that, for any game $\mathcal{G}\left(n, Q, X, Z, \Theta, \Phi\right)$ there are at least one Cournot-Nash equilibria in which $\overline{\theta}_j^i \overline{\varphi}_j^i = 0$, for any pair $(i,j) \in I \times J$. We refer to these equilibria as normalized Cournot-Nash equilibria.

Finally, the existence of an equilibrium in our economy is a consequence of the following result.

Proposition 2. *Under Assumptions (A1)-(A3), if* $(n, Q, X, Z, \Theta, \Phi) \gg (\overline{n}, \overline{Q}, \overline{X}, \overline{Z}, \overline{\Theta}, \overline{\Phi})$*, then every normalized Cournot-Nash equilibrium for* $\mathcal{G}\left(n, Q, X, Z, \Theta, \Phi\right)$ *is an equilibrium of the original economy.*

Proof. Let $\left(\overline{p}, \overline{\pi}, \overline{q}, (\overline{\eta}^i; i \in I)\right)$, where $\overline{\eta}^i = \left(\overline{x}^i, \overline{z}^i, \overline{\theta}^i, \overline{\varphi}^i\right) \in \overline{\Gamma}^i$, be a normalized equilibrium for the generalized game $\mathcal{G}\left(n, Q, X, Z, \Theta, \Phi\right)$, with $(n, Q, X, Z, \Theta, \Phi) \gg (\overline{n}, \overline{Q}, \overline{X}, \overline{Z}, \overline{\Theta}, \overline{\Phi})$.

Step I: Market feasibility. Aggregating agent's first period budget constraints we have,

$$\overline{p}_0 \sum_{i\in I} \left(\overline{x}_0^i - w_0^i\right) + \overline{\pi} \sum_{i\in I} \overline{z}^i + \sum_{i\in I}\sum_{j\in J} \overline{q}_j (\overline{\theta}_j^i - \overline{\varphi}_j^i) \leq 0.$$

It follows that, if for some commodity $l \in L$, $\sum_{i\in I}\left(\overline{x}_{0,l}^i - w_{0,l}^i\right) > 0$ then the auctioneer $h(0)$ will choose the greater price for this good, $\overline{p}_l = 1/\zeta_l > 0$, and zero prices for the other goods and assets, making his objective function positive. A contradiction. Therefore, for any commodity $l \in L$ we have that $\sum_{i\in I} \overline{x}_{0,l}^i \leq \sum_{i\in I} w_{0,l}^i < W_{0,l}$.

On the other hand, suppose that for some asset $j \in J$, $\sum_{i\in I}(\overline{\theta}_j^i - \overline{\varphi}_j^i) > 0$. Then the auctioneer $h(0)$ would choose the maximum price possible for this asset, that is $\overline{q}_j = Q > \overline{Q}$, a contradiction with the maximum price that is compatible with the existence of some agent that buys the asset (see Lemma 4 above). Thus, we conclude that, for any asset $j \in J$, $\sum_{i\in I} \overline{\theta}_j^i \leq \sum_{i\in I} \overline{\varphi}_j^i$.

Moreover, if the aggregated of risk-free asset positions is positive, i.e., $\sum_{i\in I} \overline{z}^i > 0$, then the auctioneer $h(0)$ would choose a price $\overline{\pi} = n$, which is greater than $\overline{n}$ (the maximum price possible as was proved in Lemma 3), a contradiction. Thus, $\sum_{i\in I} \overline{z}^i \leq 0$.

As we prove above, the consumption at the first period is bounded from above by the aggregate endowment at this period, which is less than the upper bound X. Thus, budget constraints at the first period are satisfied with equality and the objective function of the auctioneer $h(0)$ as an optimal value equal to zero. As a consequence, if $\sum_{i\in I}\left(\overline{x}^i_{0,l}-w^i_{0,l}\right)<0$, the auctioneer $h(0)$ would choose a zero price for the good l, a contradiction with the strictly monotonicity of preferences (Assumption (A1)). Therefore, $\sum_{i\in I}\overline{x}^i_0=W_0$.

Analogously, if $\sum_{i\in I}\left(\overline{\theta}^i_j-\overline{\varphi}^i_j\right)<0$, the auctioneer would choose a zero prices again, a contradiction since $(R_{s,j};s\in S)\neq 0$ and preferences are strictly monotonic. Finally, if $\sum_{i\in I}\overline{z}^i<0$, then the auctioneer would choose a zero price for the risk-free asset, resulting in another contradiction. Then, market feasibility holds at $t=0$ in physical and financial markets.

Using market feasibility of the allocations $\left(\left(\overline{x}^i,\overline{z}^i,\overline{\theta}^i,\overline{\varphi}^i\right);i\in I\right)$ at the first period, and aggregating budget constraints at state of nature $s\in S$ we have that $\overline{p}_s\sum_{i\in I}\left(\overline{x}^i_s-w^i_s\right)\leq 0$. Therefore, as $\overline{p}_s\cdot\zeta=1$, we have that $\sum_{i\in I}\left(\overline{x}^i_{s,l}-w^i_{s,l}\right)\leq 0$, for any $l\in L$.

It follows that budget constraints are satisfied with equality, because consumption allocations are bounded by the aggregated endowments at this state of nature and $\sum_{i\in I}w^i_s<2\overline{W}(1,\ldots,1)$. Finally, if $\sum_{i\in I}\left(\overline{x}^i_{s,l}-w^i_{s,l}\right)<0$, then the auctioneer $h(s)$ would choose a zero price for the good $l\in L$. A contradiction. We conclude that market feasibility also holds at state of nature $s\in S$.

Step II. Optimality of individual allocations. Since market feasibility holds in physical markets, it follows that $\overline{x}^i_{0,l}<X$ and $\overline{x}^i_{s,l}<2\overline{W}$, for any $(i,s,l)\in I\times S\times L$. As we have a normalized equilibrium, it follows from Lemma 2 that, for any $(i,j)\in I\times J$, $\max\{\overline{\theta}^i_j,\overline{\varphi}^i_j\}<\min\{\Theta,\Phi\}$. Also, for any $i\in I$, $\overline{z}^i\in(-Z,Z)$. Thus, for any $i\in I$ the allocation $\overline{\eta}^i$ belongs on the interior of the set $K(X,Z,\Theta,\Phi)$ with respect to $\Gamma(\mathbb{P}^i)$.

Thus, if there exists another allocation $\eta^i\in\Gamma(\mathbb{P}^i)$ such that $U^i(\eta^i)>U^i(\overline{\eta}^i)$, then for $\lambda\in(0,1)$ sufficiently small, we have that $\eta^i(\lambda):=\lambda\eta^i+(1-\lambda)\overline{\eta}^i\in K(X,Z,\Theta,\Phi)$. By the strictly concavity of utility function, we have $U^i(\eta^i(\lambda))>U^i(\overline{\eta}^i)$, a contradiction with the optimality of $\overline{\eta}^i\in\overline{\Gamma}^i$. Therefore, for any $\eta^i\in\Gamma(\mathbb{P}^i)$, $U^i(\eta^i)\leq U^i(\overline{\eta}^i)$, which proves the optimality of $\overline{\eta}^i\in B^i(\overline{p},\overline{q},\overline{\pi})$ among the allocations in the agent i's budget set. □

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