SUSTAINABLE DEVELOPMENT AND NATURAL RESOURCE ACCOUNTING IN A SMALL OPEN ECONOMY: A METHODOLOGICAL CLARIFICATION

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Abstract

The effort to correct the national accounts in order calculate NNP or related ‘Green GDP’ concepts, known as natural resources accounting, has been a lively research area in the last decade. Two basic methodologies have been proposed in the literature to value the loss of natural assets, the net price or depreciation method and the user cost approach. This paper aims to show that the user cost approach is incorrect and misleading. In arguing for its use, its original proponent implicitly had the context of a small open economy in mind. However, in this context, the depreciation method is somewhat different from its closed economy counterpart. In a small open economy, to arrive at a sustainable NNP figure changes in foreign assets must also be accounted for. Once this is done, the main criticism to the depreciation method—that resource rich countries would not have a consumption advantage over resource poor countries—can be shown to be wrong. For this and other reasons it is recommended that only the depreciation method be used in resource accounting methodologies. As a secondary result, this paper stresses the importance of incorporating changes in foreign assets in applied work on resource accounting in small developing countries.

Resumen

La contabilidad de recursos naturales –el campo de la economía ambiental que busca corregir las cuentas nacionales para obtener indicadores de desarrollo sustentable tales como el ‘PGB Verde’– ha sido un área de intensa investigación durante la última década. En esta literatura se han propuesto dos metodologías para valora las pérdidas del capital natural, el método del precio neto o depreciación y el método de costo de uso. Este trabajo intenta demostrar que el método de costo de uso es incorrecto. Al abogar por su uso, el proponente original implicitamente tenía en mente una economía pequeña abierta a la economía internacional. Sin embargo, en tal contexto, el método de depreciación requiere hacer un ajuste levemente distinto que en el contexto de una economía cerrada. En una economía pequeña y abierta, para obtener un indicador de...
1. INTRODUCTION

The effort to correct the national accounts in order calculate NNP or related ‘Green GDP’ concepts, known as natural resources accounting, has been a lively research area in the last decade. Natural resource accounting aims to provide better income and welfare measures in order to evaluate whether countries, especially resource rich ones, are on a sustainable consumption path. Within the framework of the revision of the National Accounts System, the United Nations published in 1993 a document on the integration of environmental and economic accounts and is now in the process of developing an Operational Manual. Although the United Nations recommends that environmental accounts should for the time being be registered in satellite accounts complementary to the core NAS economic accounts, there is a clear signal that natural resource and environmental accounting are making their way into the mainstream national accounting system.

One of the problems that has hindered further development of natural resource accounting—and its formalization in the core NAS accounts—is the multiplicity of methods found in the literature regarding the correct way to value losses of natural resource assets. Two basic methodologies have been proposed in the literature, the net price method (associated with the pioneering work of Repetto, McGrath, Wells, Beer, and Rossini, 1989), and the user cost approach (El Serafy, 1989).

In relation to these valuation methods, empirical work has evolved in parallel. Some studies such as Repetto, et al. (1989) for Indonesia, Solorzano, et al. (1991) for Costa Rica, and Young (1993) for Australia, have used the net price method, while other studies have used the user cost approach3.

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1 See, for example, the special issue of Environment and Development Economics in the 2000 February/May volume, dedicated to natural resource accounting.
It is interesting to note that some studies, such as van Tongeren, et al. (1993) for Mexico, Bartelmus, et al. (1993) for Papua New Guinea, Young (1992) for Brazil use both methods and compare results.

This methodological dichotomy is certainly puzzling. Authors such as Bartelmus (1999), from the United Nations Statistical Office, as well as others still consider both methods as distinct and alternative valuation techniques. The purpose of this paper is to clarify this issue by showing that the user cost approach is incorrect and misleading. In arguing for its use, its original proponent implicitly had the context of a small open economy in mind. However, in this context, the depreciation method is somewhat different from its closed economy counterpart. In a small open economy, to arrive at a sustainable NNP figure changes in foreign assets must also be accounted for. Once this is done, the main criticism of El Serafy (1989) to the depreciation method—that resource rich countries would not have a consumption advantage over resource poor countries—can be shown to be wrong. In addition, the user cost approach is an ad-hoc method that is not grounded on an optimal resource extraction model. For these reasons it is recommended that only the depreciation method be used in resource accounting methodologies.

As a secondary result, this paper stresses the importance of incorporating changes in foreign assets in applied work on resource accounting in small developing countries. The recent experience of countries such as Argentina shows that unsustainable foreign borrowing can be a major obstacle to future consumption growth and thus a crucial factor in any measure such as green NNP that attempts to gauge the future prospects for economic wellbeing. Important recent studies, such as Vincent (1997) for the Malaysian economy, ignore these changes, which decreases the usefulness of these empirical results as a measure of future consumption growth.

2. The Measurement of Sustainable Growth

As stated in the introduction, natural resource accounting aims to provide indicators of the sustainability of current economic activity. The basic idea is to be able to compare current consumption, \( C(t) \), to a measure of sustainable consumption, \( C_s(t) \) in order to evaluate whether the present generation is living “beyond its means”. \( C(t) > C_s(t) \) would indicate that somehow future wellbeing is being jeopardized by an excessively high current consumption level. Although the measurement of \( C_s(t) \) should be of general interest to national income accountants, the impetus for research in this area during the last decade has been motivated by the growing concerns regarding the environment and the use of natural resources. It has been widely recognized that traditional measures of economic activity, such as GDP, are particularly flawed with respect to the treatment of natural resources and the environment. Depletion of these assets is implicitly treated as consumable income in traditional national income figures.

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4 Another example is Perman, et al. (1999), a popular text on environmental economics, which presents both methods as alternative valuation techniques in a chapter on natural resource accounting.
This raises the specter that unsustainable consumption levels, based on the running down of these natural assets, will not be detected in time to avoid detrimental effects on future generations.

Although the basic idea of natural resource accounting is clear, two questions must be addressed in theory and in practice. First, what exactly is $C_s(t)$? Second, how can it be measured and what relation does it have to traditional national income figures? The first question is relevant because there are at least three ways to define $C_s(t)$ (Asheim, 2000). First we could define $C_s(t)$ as the constant consumption level that generates the same discounted welfare level as the true consumption path of an economy, $C(s)$:

$$\int_{t}^{\infty} U(C(t))\lambda(s)ds = \int_{t}^{\infty} U(C(s))\lambda(s)ds$$

where $U(.)$ is a utility function, and $\lambda(s)$ is a utility discount factor. This measure is called ‘welfare equivalent income’, and a comparison of current consumption with that measure would indicate how the welfare of the current generation compares with the average welfare of future generations. Alternatively, we could define $C_s(t)$ as the constant consumption level that provides the same discounted total consumption as the true consumption path of the economy:

$$\int_{t}^{\infty} C(t)\delta(s)ds = \int_{t}^{\infty} C(s)\delta(s)ds$$

where $\delta(s)$ is now the consumption discount factor. This measure is called ‘wealth equivalent income’ and comparing it to current consumption would indicate how the consumption of the current generation is related to the weighted average consumption of the future generations. These two measures do not coincide except when utility is linear in consumption. Moreover, neither of these consumption measures are feasible when technology is convex. This gives rise to a third definition of $C_s(t)$, ‘sustainable income’:

$$\text{Max } C(t) \text{ such that } C(s) = C(t) \text{ is feasible}$$

Sustainable income is the maximum amount of consumption that can be enjoyed this period and enjoy the same constant amount in the future.

Which sustainability measure should be used? Fortunately, in the context of a small open economy, the last two measures (wealth equivalent income and sustainable income) coincide. In addition, Asheim (2000) recommends the use of these last two measures since they have a relation to national accounting indicators based on current quantities and prices. Therefore, for the remainder

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5 See also the excellent introduction by Vincent (2000).

6 Alternatively one could assume that the utility of each generation is linear in consumption.
of this paper we will consider sustainable income or wealth equivalent income as our sustainability criteria. A second question is how we can obtain such a measure in practice. This is the topic of the next section.

3. HARTWICK’S RULE IN A SMALL OPEN ECONOMY

The approach of this section is based on Solow (1986) and Hartwick (1990). They use Weitzman’s (1976) result, which states that NNP is equal to the current value Hamiltonian of the corresponding optimal growth problem. In an economy with an exhaustible resource the Net National Product will be equal to GNP minus the Hotelling rents from natural resource extraction. Solow (1986) shows that if Hartwick’s rule—reinvest all Hotelling rents from resource extraction—is imposed on such an economy then consumption will be constant. Therefore, NNP is the income measure we are seeking.

Hartwick’s rule was developed in a closed economy context. This is reflected by the exclusion of foreign trade and foreign assets. In a comment to Solow (1986), Svensson notes that:

“For an open economy, intertemporal trade, i.e., borrowing and lending, implies that consumption need not equal output of the consumption good at each point in time. In particular, for a small open economy, the interest rate is given by the world capital market, and consumption and investment decisions become separable. A small open economy should simply choose investment so as to maximize wealth. If there is a social preference for a constant consumption path, it can simply be chosen subject to an intertemporal budget constraint, and is otherwise completely independent of the specific investment and production path. These circumstances combined make me believe that Hartwick’s rule, although a very neat theoretical result, is of limited interest for discussing intergenerational equity in small open economies.”

I will try to show that in a small open economy Hartwick’s rule of reinvesting all Hotelling resource rents is still valid. However, an allowance has to be made for the change in foreign assets.

Just like in the original sustainability literature, the following discussion is couched in terms of an exhaustible resource. The model developed below can and has been extended to allow for unanticipated and anticipated discoveries of the exhaustible resource, and for the case of renewable resources. Since these extensions do not alter the main points presented in this paper, the simpler standard exhaustible resource model without discoveries is used. In addition, our model assumes there is no technical change or population growth.

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7 For simplicity we assume that the man-made capital stock does not depreciate. If this was not so the depreciation of this stock would also have in be deducted from GNP to arrive at a NNP figure.

8 For extensions along these lines see Weitzman (1997) and Asheim (1997).
The extension to an open economy is straightforward. The maximization problem is,

\[ \text{Max} \int_{0}^{\infty} e^{-\delta t} U(C(t))dt \]

subject to:

\[ \dot{A} = X - M + rA = Y(K, R) - C - I - f(R, S) + rA \]

\[ \dot{K} = I \]

\[ \dot{S} = -R \]

where \( A \) is the stock of foreign assets, \( X \) is exports, \( M \) is imports, \( r \) is the international interest rate (exogenous for this small economy), \( Y(K, R) \) is the production function of the composite good with the capital stock, \( K \), and resource extraction, \( R \), as arguments (labor is assumed fixed), \( C \) is the aggregate consumption, \( I \) is investment, \( f(R, S) \) is the cost of extraction (measured in terms of the composite good), \( S \) is the stock of the natural resource, and \( \delta \) is the discount rate.

The current value Hamiltonian for this problem is,

\[ H = U(C) + \lambda_1(I) + \lambda_2(-R) + \lambda_3(Y(K, R) - C - I - f(R, S) + rA) \]

and the corresponding first order conditions are,

\[ \frac{\partial H}{\partial I} = \lambda_1 - \lambda_3 = 0 \]

\[ \frac{\partial H}{\partial R} = -\lambda_2 + \lambda_3(Y_R - f_R) = 0 \]

Asheim (1986), Asheim (1994) and Hartwick (1995) examine the issue of constant consumption paths in open economies with exhaustible resources. However, these articles are concerned with terms of trade effects on consumption paths and net national product measures. In the present article, a small open economy context is assumed, where world prices and interest rates are exogenous to the country in question. The reason for this is that it is the context implicit in the El Serafy (1989) user cost approach. Hartwick (1995) presents a model of an oil exporter facing constant prices and interest rates and derives very similar results to ours. However, the present model is more general and is more focused to comparing the depreciation method with the user cost approach. See also Vincent, Panayotou and Hartwick (1997).

Time subscripts have been omitted to simplify notation. However, except for the discount rate and the intentional interest rate, which are assumed constant, each variable above is a function of time.
(8) \[
\frac{\partial H}{\partial C} = U_C - \lambda_3 = 0
\]
(9) \[
\dot{\lambda}_1 = \delta \lambda_1 - \lambda_3 Y_K
\]
(10) \[
\dot{\lambda}_2 = \delta \lambda_2 - \lambda_3 f_S
\]
(11) \[
\dot{\lambda}_3 = \delta \lambda_3 - \lambda_3 r
\]

Substituting (6), (7) and (8) into (5) yields the value of the Hamiltonian in an optimal path expressed in utility units,

(12) \[
H = U(C) + U_C I - U_C (Y_R - f_R) R + U_C Y(K,R) - C - f(R,S) + rA
\]

Using a linear approximation to utility, \(U(C) = U C\), and dividing (12) by \(U(C)\), we get a monetary value of the Hamiltonian in an optimal path\(^{11}\), or the Net National Product,

(13) \[
\frac{H}{U_C} = NNP = C + I - (Y_R - f_R) R + \dot{A}
\]
(14) \[
NNP = C + I - (Y_R - f_R) R + X - M + rA
\]
(15) \[
NNP = GDP - (Y_R - f_R) R + rA
\]

This result is similar to the closed economy case. To derive NNP, Hotelling resource rents have to be deducted from GDP, but, in addition, the interest earned on the foreign asset stock has to be added\(^{12}\). In most developing countries, where the small open economy hypothesis is relevant, \(A\) will be negative due to net foreign indebtedness. Thus, to arrive at NNP foreign debt services have to be deducted from GDP.

This result is intuitively simple and is not new. Solow (1986) expresses Hartwick’s rule in a setting with multiple types of capital as \(p(t)(\delta K/\delta t) = 0\), where \(p(t)\) is a vector of shadow prices for investment goods and \((\delta K/\delta t)\) is a vector in which each element is the change in one type of capital. Since foreign assets are just another type of capital, the change in this asset has to be accounted for. Since the traditional measure of GDP already includes foreign trade, only a correction for interest earnings or payments has to be made.

To anyone familiar with national accounting concepts it will be immediately clear that the above procedure is equivalent to deducting resource rents from GNP, since this last figure includes foreign assets interest payments. In fact Hartwick (1990) correctly considers GNP as the measure that has to be

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\(^{11}\) See Weitzman (2000) for a justification for this approach.

\(^{12}\) The adjustment to GDP for the change in foreign assets is limited to the interest earned on these assets only. The other components of the change in foreign assets, exports minus imports, are already accounted for in the traditional measure of GDP.
modified to arrive at NNP. However, in this paper the role of foreign assets is stressed for several reasons.

First, empirical studies such as the seminal work by Repetto, *et al.* (1989) use GDP instead of GNP, perhaps because the statistical offices of developing countries do not calculate GNP figures as often as GDP. In many of these countries, the change in foreign assets due to excessive indebtedness may be just as dangerous to sustainability as the loss of natural assets. Therefore, it is important to stress that the change in foreign assets must be explicitly accounted for when deriving sustainability indicators for a developing economy.

Second, by formulating the role of foreign assets explicitly in the above optimization problem, some insight is gained as to the difference between the depreciation method and the user cost approach. This will be made clear in the next section.

Following Solow (1986) it can be shown that an extended Hartwick rule (invest in reproducible capital an amount equal to the Hotelling rents from resource extraction minus the change in foreign assets) will ensure a constant consumption path for the small open economy case. This is shown in the appendix of this paper.

Before continuing it must be stressed that the above result pertaining to the constancy of consumption is altered if there are anticipated exogenous changes to the terms of trade of the country. In this case, following Hartwick’s rule will not result in a constant consumption path. However, the necessary adjustments to the rule required in order to guarantee a constant consumption path have been derived in the literature (Hartwick (1995); Vincent, Panayotou and Hartwick (1997)). Moreover, changes in foreign assets must still be account for in the sustainability measure.

### 4. The Different Approaches to Natural Resource Accounting

The result in the previous section shows that to arrive at the Net National Product all Hotelling rents from resource extraction have to be deducted from GDP (as well as interest earnings or payments of foreign assets). This is consistent with the depreciation or net price approach to natural resource accounting as in Repetto, *et al.* (1989). However, El Serafy (1989) criticizes the depreciation method and instead proposes an alternative approach whereby a fraction of current total (as opposed to Hotelling) rents are deducted from GDP to arrive at a sustainable growth indicator.

El Serrafy’s main criticism to the depreciation approach is that since all resource rents are deducted form GDP, a country with a large endowment of

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13 Hamilton and Clemens (1999) do take into account changes in foreign assets in their calculations.

14 For the case of endogenous terms of trade changes when the country is not small relative to the world economy see Asheim (1986; 1994).

15 Another question is what happens when there are unanticipated changes in the terms of trade. Presumably, current consumption and NNP would jump to adjust to the new wealth level implicit in the new terms of trade. But, thereafter, the same sustainability accounting as derived here would have to be followed.
natural resources would not seem to have an income (i.e. permanent consumption) advantage over other countries\textsuperscript{16}. This result would obviously be flawed.

As an alternative, El Serafy (1989) proposes the following\textsuperscript{17},

\begin{equation}
\int_{0}^{\infty} e^{-rt} I dt = \int_{0}^{\infty} e^{-rt} R(0) dt
\end{equation}

where $I$ is income, $R(0)$ is the rent generated in the current period from resource extraction, $n$ is the number of years that the resource will last given a constant extraction rate equal to the current rate, and $r$ is an exogenous interest rate.

Equation (16) transforms a finite cash flow (from resource extraction rents) to an infinite income flow. Since $I$ and $R(0)$ are constant we can integrate both sides of (16) and arrive at,

\begin{equation}
\frac{I}{R(0)} = \left(1 - \frac{1}{e^n}\right)
\end{equation}

Equation (17) gives the proportion of current rents (if maintained for $n$ years) that can be transformed into income and thus consumed. Therefore, it would seem that contrary to what was derived in the previous section, it is not Hotelling rents that must be deducted but only a fraction $(1/I/R(0))$ of total rents\textsuperscript{18}. Thus, the result that to arrive at NNP Hotelling rents should be deducted from GNP would seem to be incorrect.

Before discussing El Serafy’s approach further we will pursue a point that was not mentioned in the optimal control problem of section 3. Namely, that there is an additional restriction to that optimal control problem that might shed some light as to the reason why El Serafy’s method differs from the results of that section.

What is the maximum constant sustainable consumption that an economy characterized by equations (1) to (4) may enjoy? As Svensson mentions: “...it can simply be chosen subject to an intertemporal budget constraint, and is otherwise completely independent of the specific investment and production path”.

The intertemporal budget constraint is given by the following condition\textsuperscript{19},

\begin{equation}
\lim_{t \to \infty} e^{-rt} A(t) = 0
\end{equation}

which states that in the long run a country cannot be a net lender or borrower in present value terms. This implies that a country’s foreign debt should not grow faster than the interest rate it has to pay.

\textsuperscript{16} On this point, El Serafy seems to be confused between total rents and Hotelling rents. Only the later have to be deducted from GDP according to the depreciation method. If there are increasing extraction costs, total rents will be larger than Hotelling rents.

\textsuperscript{17} El Serafy (1989) derives his formula in discrete time. However, its continuous time analogue is used here.

\textsuperscript{18} Vincent (1997) derives a similar formula to El Serafy’s but by another method. His interest is to approximate Hotelling rents using total rents.

\textsuperscript{19} The following derivations follow Sachs (1982) very closely.
The first restriction of the optimization problem (equation (2)) was,

$$\dot{A} = X - M + rA = Y(K, R) - C - I - f(R, S) + rA$$

Integrating this differential equation yields,

$$A(t) = A(0)e^{rt} + e^{rt}\int_{0}^{t} e^{-r\xi} (Y(K(\xi), R(\xi)) - C(\xi) - I(\xi) - f(\xi))d\xi$$

and using the limit condition (18) yields,²⁰

$$\int_{0}^{\infty} e^{-rt} (Y(t) - C(t) - I(t) - f(t))dt = 0$$

This last equation is the intertemporal budget constraint. It states that the present value of trade deficits has to equal the present value of trade surpluses.

We can transform equation (20) into an alternative form that will be more useful. Let us define the “permanent” or “perpetual equivalence” of a variable as,

$$\int_{t}^{\infty} e^{-r(\tau-t)} X^P(\tau)d\tau = \int_{t}^{\infty} e^{-r(\tau-t)} X(\tau)d\tau$$

$X^P$ is the constant value of the variable that over the horizon will give the same present value as the changing variable path $X(\tau).²¹$

Defining $Y^P(0), C^P(0), I^P(0), f^P(0)$ as the permanent values of $Y(t), C(t), I(t)$ and $f(t)$ respectively, the budget constraint (20) after integrating is,

$$\frac{C^P(0)}{r} = \frac{Y^P(0) - I^P(0) - f^P(0)}{r} = \text{Wealth}$$

or

$$C^P(0) = r\text{Wealth}$$

Therefore the maximum consumption feasible in this economy is the interest from the wealth that is possible to generate with the initial endowments.

Now we are ready to discuss El Serafy’s formula. Implicitly, his method is only valid for a small open economy, because the interest rate is exogenous. In addition, the flavour of his argument implies a small open economy since the intuition behind his result is that part of the resource rents can be invested in the international market and the return on these assets will be used to finance consumption after the resource is exhausted.

²⁰ For simplicity it is assumed that $A(0) = 0$.
²¹ As an example, Weitzman (1976) showed that NNP at time $t$ is the perpetuity equivalent consumption of the optimal and variable consumption path.
From the previous discussion it should be clear that El Serafy’s calculation of “consumable” income, \( I \), is just the perpetuity equivalent of the wealth generated by the natural resource rents. This is what equation (16) does. Therefore El Serafy is correct in the sense that consumption is a fraction of total wealth. However, in the small open economy case, consumption and investment decisions are independent. Therefore the level of consumption is independent of the amount of resource rents generates in the current period. The extraction path of the resource should be such that wealth is maximized.

We have seen that in an efficient path to construct an NNP indicator we have to deduct Hotelling resource rents from GDP as well as any foreign debt interest payments. In the case where the country is exploiting its resource base to built up foreign assets, \( rA \) will be positive. Therefore the amount of investment necessary to maintain a constant consumption path is less than the total resource rents of the period, confirming El Serafy’s intuition. However, the correct way to account for such an effect is to deduct all resource rents and then incorporate the change in foreign assets.

Since El Serafy (1989) does not postulate a behavioural model of resource extraction it is not clear why rents are generated in his formulation in the first place\(^{22}\). Therefore it is not possible to see whether the correcting factor derived from our optimization model, \((Y_f - fR)R + rA\), would be equal to the depreciation allowance of El Serafy method \((R(0)/e^n)\). We do know however that in an optimal growth path, where resource extraction is efficient, NNP is equal to GDP minus resource rents and net interest payments.

The main thrust of El Serafy’s criticism of the depreciation approach is no longer valid. Deducting rents from GDP will not reduce NNP to zero since the current account will adjust to make NNP equal to permanent consumption (if Hartwick’s extended rule is imposed).

5. CONCLUSIONS

The conflict between sustainable development and optimal economic growth is really a problem of how to measure growth. In this paper an approach to measuring national income for a small open economy was discussed. The result shows that the two principal methods for accounting natural resource depreciation are not valid in a small open economy context. However, the depreciation approach would be the correct method if it is extended to include interest earnings or payments on foreign assets.

There are many other topics in natural resource accounting that merit discussion. One empirically relevant point is the correct way to account for discoveries. Hartwick (1994) includes a discovery function in the optimal growth problem and concludes that the correct depreciation measure is the value of the net change in assets (extraction minus discoveries).

\(^{22}\) His assumption of a constant rent stream \( R(0) \) for \( n \) periods is a highly unlikely outcome of any model in which resource owners are somehow optimizing intertemporally. If they are not optimizing intertemporally then resource rents may not arise in the first place.
One final point worth mentioning is that the models used to derive constant consumption paths are extremely simplified versions of an economy and therefore care has to be taken when extracting policy implications. In particular, if there is technological change then Hartwick’s rule is too conservative as a way of guaranteeing intergenerational equity. Hartwick’s rule, more than a precise theoretical result, should be viewed as a rule of thumb to help us think about sustainability. In Solow’s words: “... I could see the rule as a rebuttable presumption, as a way of constantly reminding ourselves that there are considerations other than immediate utility to be taken into account” (Solow, 1986).

REFERENCES


APPENDIX

The following appendix shows that the extended Hartwick rule implies a constant consumption path. To see this we use Weitzman’s result regarding the welfare meaning of NNP. Weitzman (1976) shows that the net present value of the optimal consumption path is exactly equal to the net present value of a constant consumption stream equal to the current NNP. In other words, NNP in period $t$ is the perpetual equivalence of the optimal consumption path from $t$ onwards. Mathematically,

$$
\int_{t}^{\infty} e^{-\delta(s-t)} NNP(t) ds = \int_{t}^{\infty} e^{-\delta(s-t)} C^*(s) ds
$$

where $NNP(t)$ the current period Net National Product and $C^*(s)$ is the consumption level at each moment of time given by the optimal growth path.

The extended Hartwick rule would be,

$$
I = (Y_R - f_R) - \dot{A} = (Y_R - f_R) - (X - M + rA)
$$

By equation (14) we see that if the extended Hartwick rule is imposed on the economy in period $t$ then $NNP(t)$ will just be the consumption level $C^*(t)$. Therefore we can write equation (A.1) as,

$$
C^*(t) = \delta \int_{t}^{\infty} e^{-\delta(s-t)} C^*(s) ds
$$

taking the derivative of equation (A.3) with respect to $t$ we have,

$$
\frac{\partial C^*(t)}{\partial t} = \delta \left[ -\delta e^{\delta t} \int_{t}^{\infty} e^{-\delta s} C^*(s) ds + e^{\delta t} \left( e^{-\delta t} C^*(t) \right) \right]
$$

$$
\frac{\partial C^*(t)}{\partial t} = \delta \left[ C^*(t) - C^*(t) \right] = 0
$$

Therefore, the extended Hartwick rule guarantees that consumption is constant over time. If production possibilities are convex however, this constant consumption level is not feasible in a closed economy (see Asheim, 2000). However, in an small open economy with given terms of trade, this consumption level is feasible.