The tax paradox and weak tax neutrality

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Abstract. We introduce the concept of weak tax neutrality which establishes that the relationship between the tax rate and the user cost of capital may be non-monotonic. We show that most existing corporate tax systems allow for weak neutrality. That is, given the tax allowances permitted by these systems, it is possible that neutrality may arise for at least one positive corporate tax rate. Moreover, we show the practical relevance of weak neutrality in realistic situations where there are several asset types and heterogeneous levels of firms’ debt ratios.

Key words: Tax code neutrality, corporate profit tax, optimal taxation, non-distortionary tax systems, rent taxation.

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1. Introduction

The effect of corporate taxes on investment is a major issue in public finance; it is relevant in a theoretical venue as well as for the design of efficient tax policies. A key concern regarding corporate taxes is that they may negatively affect firms’ investment incentives. This has led to the development of various tax methods, which essentially use economic rents as the taxable income. These methods are strongly neutral in the sense that any positive tax rate between 0 and 1 may cause no distortion on the investment choice.

The best-known strongly neutral methods are Samuelson’s (1964) Imputed Income method (IIM) and Brown’s (1948) Cash Flow method (CFM). The IIM allows firms to deduct from the firms’ taxable earnings the true capital depreciation over time as well as the imputed interest costs of the new investment, while the CFM permits full and instantaneous depreciation of investments at the time they are implemented. However, most tax systems

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\(^2\) Important theoretical studies are Hall and Jorgensen (1967); Tobin (1969); Abel and Eberly (1994); Alvarez et al. (1998); Dievert and Lawrence (2002) among others. An early survey can be found in Mintz (1995) and Hasset and Hubbard (2002) summarizes newer empirical literature. More recent studies are Djankov et al. (2010), Ljungqvist and Smolyansky (2014) and Zwick and Mahon (2017).

\(^3\) Ruf (2012) shows that the taxable base attained from Samuelson’s method can be enhanced by forcing all pure profits to be part of the tax base.

\(^4\) More recently, Boadway and Bruce (1984) have shown that allowing for the true capital depreciation is not necessary for neutrality as long as the allowed depreciation rates mimic the true ones in terms of present value. Also, Devereux and Freeman (1991) propose the ACE (Allowance for Corporate Equity) method, which allows firms to deduct a notional interest rate on their equity, and Bond and Devereux (1995) provide some generalizations of the theoretical background underlying the ACE method. In addition, in 1992, the U.S. Treasury Department proposed the CBIT (Comprehensive Business Income Tax) method, which does not allow deductions of either interest payments or the return on equity from taxable corporate earnings. These methods are also strongly neutral by permitting any corporate profit tax rate between 0 and 1 to have no effect on the user cost of capital (de Mooij and Devereux (2011)). Another interesting example of a strongly neutral tax system is given by Alvarez and Kanniainen (1997). However, Niemann (1999) proves that their results have some relevant limitations.
in the world do not satisfy the conditions required for the application of these methods (Bond and Xing, 2015).

Here, we consider the case of a corporate tax system that satisfies the following two highly prevalent features among tax codes around the world: (a) it provides at least a partial allowance for capital depreciation costs (for example, in the form of accelerated depreciation allowances), in combination with, (b) partial or full tax allowance for the interests paid on the portion of the investment that is financed with debt. We denote this as the “classical tax system” (CTS).  

The major point of this paper is that corporate taxes do not need to be applied using strongly neutral tax methods to be neutral. Under certain conditions often satisfied by CTS, it is possible to determine at least one positive corporate tax rate which is neutral in the sense that it does not distort investment choices by a firm. Or, equivalently, in the general case of asset heterogeneity, there are conditions under which an existing positive tax rate may be rendered neutral by adjusting the asset depreciation tax allowances. Because neutrality can be achieved only for some tax rates and not for any tax rate as in the case of the strongly neutral methods, we denote this method weakly neutral. To show this point with as little algebraic clutter as possible, we first use the simplest model of investment for a representative firm available in the literature which focuses on debt as a key source of investment finance (Hall and Jorgenson, 1967).

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5 In fact, the corporate tax base in almost all OECD countries corresponds to a measure of company profits net of allowances for interest payments and presumed depreciation costs (MIRRLESS et al. 2011, chapter 17). As de MOOIj and Devereux (2009, page 9) explain, “Most corporate tax systems in the world allow interest to be deductible as expenditure when calculating taxable profits. The normal returns on equity are usually not deductible as a cost.” These are the two key characteristics of the tax codes we consider in our analysis.
The contribution of our work is based on a concept that was originally described by Schneider (1969) called ‘taxation paradox’, who argues that there may exist an interval where the cost of capital is decreasing with the tax rate (see also Hall and Jorgenson, 1971, King, 1977, Steiner, 1980, Bustos et al. 2004 and, more recently, Alvarez and Koskela, 2008 and Gries et al. 2012). In a different context, studies showed that the cost of capital could decrease with an increase in the tax rate within certain ranges of the tax.

However, we go beyond simply corroborating the existence of a tax paradox and show that, under certain plausible conditions, the relationship between the tax rate and the user cost of capital may be U-shaped. It is the existence of such U-shaped relation which permits the existence of weak neutrality. In other words, we show that the existence of the tax paradox is a necessary but not sufficient condition for weak neutrality. And it is the potential for weak tax neutrality that gives most policy relevance to the tax paradox.

The literature has not yet developed a formal analytical solution to show the conditions for the existence of a positive tax rate that achieves neutrality and study the determinants of such tax if the conditions for existence are met. The present work is a first attempt to achieve these objectives.

We first show the analysis for a representative firm investing in just one asset. Later we generalize this by allowing different kinds of assets characterized by different rates of

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6This concept was later elaborated by Sinn (1987, pp. 145-153) who argues that “an increase in the tax rate of retained profits may induce the firm to employ a higher stock of capital!”. This was consistent with the empirical findings of Jorgenson and Hall (1971) who found that a reduction in the corporate tax rate in the US from 52% to 48% increases the cost of capital.

7Since the user cost of capital may be decreasing over a range of tax rates but not for all tax rates, the tax paradox necessarily implies a non-monotonic relationship between the user cost and the tax rate. Non-linear tax schemes generate interesting policy results, see for instance Majd and Myers (1985); Mackie-Mason (1990); Sarkar and Goukasian (2006) among others.
economic depreciation. While asset heterogeneity would require using one “optimal” tax rate for each asset, something that obviously would not be plausible, we consider a different strategy. It consists in determining an optimal combination of asset depreciation tax allowances that would cause a positive tax rate to be neutral. We ascertain the conditions under which a positive tax rate may be rendered optimal by choosing a suitable combination of the tax allowances often used by CTS which are specific to each asset type considered.\(^8\) This approach is in practice feasible because most existing tax systems do allow for depreciation allowances that are in fact different for each type of asset.

Apart from the asset structure, another source of heterogeneity is related to the firms’ debt differentials.\(^9\) This implies that the optimal structure of tax allowances should also consider firms debt ranges. That is, the optimal tax allowance structure (for a given positive tax rate) would be determined for each asset type and firm’s debt type.\(^10\)

It is worthwhile to compare our results with the important recent contribution by Gries et al. (2012). Their analysis may be regarded as more general than ours in the sense that they consider uncertainty and irreversible investment while we use a more conventional deterministic model. They also show that, at the project level, there may exist an optimal

\(^8\) We note that the asset depreciation tax allowances do not in general correspond to the actual depreciation rate of the asset.

\(^9\) Apart from asset and firms’ debt heterogeneity there are other sources of heterogeneity. As recent literature showed, there are heterogeneous responses in claim tax refund for tax loses (Mahon and Zwick, 2017), heterogeneous effects in investment (Fatica, 2017a) and heterogeneous effects of the reaction between the statutory tax rate and the effective tax rate (Gemmel, Kneller, McGowan, Sans and Sanz-Sanz, 2018 and Mahon and Zwick, 2017).

\(^10\) Currently, most tax systems do consider a large number of asset depreciation allowances. The proposed weakly tax system would only involve adjusting the structure of asset depreciation allowances used to render an existing tax rate optimal. So applying a weak neutrality method does not require a drastic restructuring of the whole tax code as would be needed to implement a global neutrality method.
or neutral tax. However, the natural complexity of their model does not allow them to derive closed-form solutions for the optimal tax or for the optimal combination of tax allowances that allow a positive tax rate to be neutral, as we can do given our simpler model. This provides a potential policy relevance to the neutral tax. There are also subtle differences between our concept of neutrality and theirs that may nonetheless be conceptually important. Neutrality in our definition occurs if there exist a positive tax level that replicates the same investment level that would occur if all taxes were zero, while for Gries et al. (2012) neutrality exits if a marginal change in the tax rate does not affect the investment threshold (the point at which the project investment is triggered). Within our framework the marginal effect of the tax rate on the cost of capital at its optimum should be positive, not zero.

Finally, we do not advocate a weakly neutral method; in fact, any of the strongly neutral methods discussed earlier should be preferred to the weakly neutral one if the political conditions permit a drastic restructuring of the tax system. However, this restructuring of the tax system is often highly politically contentious, something that is corroborated by the fact that an extremely small number of countries have in fact changed their system towards one that is consistent with strong neutrality; moreover, even among some of the countries that have made the required tax reforms, these reforms have not persisted in time. In addition, the study of weakly neutral tax systems helps to understand the corporate tax for developing countries where the corporate tax revenues are more important than personal tax revenues (Baker, 2018).
Formal definitions

We first provide a formal definition of the concepts of strong and weak neutrality.

**Definition 1.** Define a tax system conformed by the corporate net income tax $\tau$ and a subset $\mu = \{\mu_i\}_{i=1}^n$ of non-tax rate instruments (investment allowances, debt interest deductions, accelerated capital depreciation, etc.). The tax system is **strongly neutral** if and only if all the possible values of $0 < \tau \leq 1$ are mapped by the following correspondence:

$$\Omega(\mu): \{\tau \in (0,1]: \varphi(\tau; \mu) = \varphi^*\} = (0,1]$$  \hspace{1cm} (1)

where, $\varphi(\tau; \mu)$ is a distorted function of some variables that depends on the tax system (i.e., the cost of capital) and $\varphi^*$ is the social optimal value of such variable.

**Definition 2.** The tax system is **weakly neutral** if and only if at least one non-zero $\tau = \tau^*$ is mapped by the following correspondence:

$$\Omega(\mu): \{\tau^* \in (0,1]: \varphi(\tau^*; \mu) = \varphi^*\} \neq \emptyset$$  \hspace{1cm} (2)

That is, there is at least one value of the corporate tax rate greater than zero which makes the undistorted value $\varphi^*$ equal to the distorted one.

**Definition 3.** Let $c(\tau, \mu)$ be the after-tax cost of capital. Assume that this function is twice differentiable. Then, there exist a **tax paradox** if $\frac{\partial c(\tau, \mu)}{\partial \tau} < 0$ for some values of $\tau$.

### 2.1 Weak neutrality: The basic theoretical model

Here we first consider the derivation of an optimal tax for a single firm using just one asset. In the following sections we generalize the analysis to allow for firms investing in more than...
one asset. Consider the following generalization of the capital arbitrage condition proposed originally by Hall and Jorgensen (1967),

\[ q_h(t) = \int_t^\infty [(1 - \tau)c_h(s)e^{-(\delta + i_h)(s-t)}]ds + \tau \chi_h q(t) \]  

(3)

Where \( q_h(t) \) is the relative market price of the capital good at acquisition time \( t \) for firm \( h \); \( 0 \leq \tau \leq 1 \) is the corporate tax rate; \( c_h(s) \) is the user cost or rental price of capital at time \( s \) (equal to the cost of capital services); \( i_h \) is the interest rate faced by the firm \( h \) after the tax allowances to be defined below; \( \delta \) is the true rate of depreciation of the asset which is assumed constant and exponential; and, \( 0 \leq \chi_h \leq 1 \) is the present value of the depreciation allowance as a proportion of the investment value for firm \( h \). As in Hall and Jorgensen (1967) we assume that the market price of the capital good is exogenous and given.

The intuition behind equation (3) is that in equilibrium, the price of one unit of capital net of tax discounts, \((1 - \tau \chi)q\), should be equal to the after-tax present value of the rental values of capital. That is, the after-tax marginal cost of one unit of investment should be equal to the marginal benefit of renting that capital unit until asset depletion.

We assume that a maximum proportion \( \eta \) of the interest costs paid by the firm can be tax-deductible and that the firm fully uses this benefit. We also assume that a proportion \( \beta_h \)
of the investment cost is financed through borrowing by firm h. For simplicity we consider $\beta_h$ as exogenous.\textsuperscript{11} This implies that the net interest rate $i(t)$ is:

$$i_h(t) = (1 - \beta_h)r(t) + \beta_h(1 - \tau \eta)r(t) = (1 - \eta \beta_h \tau)r(t). \quad (4)$$

With $r(t)$ the interest rate before tax allowances.\textsuperscript{12} Replacing (4) in (3) and differentiating with respect to time (see appendix for derivation), we get: \textsuperscript{13}

$$c_h = q[1 - \tau \chi_h \frac{(r(1 - \eta \beta_h \tau) + \delta)}{(1 - \tau)}] \equiv \varphi(\tau; \mu) \quad (5)$$

Next, we present the following condition,

**Condition A:**

$$\chi_h > \frac{(1 - \eta \beta_h) r + \delta}{r + \delta}$$

Condition A is a characteristic of the corporate tax system that implies that the depreciation tax allowance is sufficiently accelerated. This condition is sufficient for the existence of a tax paradox, that is, an increase of the corporate tax may raise the level of investment of the firm.

To understand Condition A, assume that $\beta_h = \eta = 1$. Then we have that condition A implies that $\chi_h > \frac{\delta}{r + \delta}$, where $\frac{\delta}{r + \delta}$ is the present value of the economic depreciation. Thus, the mathematical intuition of condition A is that the tax-allowed capital depreciation rate should be higher than the economic depreciation.

\textsuperscript{11}This assumption has been widely used in the literature (see for instance, Boadway and Bruce (1984), Bond and Devereux (1995) and Bustos et al, (2004), among others). We later do consider endogenous debt rates.

\textsuperscript{12}In general, we consider that firms can deduct nominal financial costs. Thus, the relevant tax rate is the nominal tax rate.

\textsuperscript{13}Note that if $\tau = 0$, then $c_h = q(r + \delta)$, which corresponds to the free market rental price of capital.
The economic intuition of condition A is that it reflects a second best problem: the tax distortion should be compensated by other distortions, in this case, the depreciation and interest deduction allowances. If the tax distortion cannot be compensated for any tax rate then no tax paradox is possible, or equivalently, there is no space for weak neutrality. Or, equivalently, Condition A would not be satisfied. That is, in this case there would be no value of $\chi_h \leq 1$ which could compensate for the tax distortion.

2.2 Basic results

Using (5) we can state the following proposition:

**Proposition 1.** Assume that Condition A is satisfied. Then the tax system is weakly neutral.

**Proof.** Using definition 2 we find a positive tax rate $\tau$ that makes the after-tax user cost of capital equal to the undistorted or free market rental price of capital; that is,

$$c(\tau; (\chi, \eta \beta)) = q \left[ 1 - \tau^* \chi_h \right] \frac{\left( r(1-\eta \beta_h \tau^*) + \delta \right)}{(1-\tau^*)} = \varphi^*. \quad (6)$$

where $\varphi^* = q(r + \delta)$.\(^{14}\)

Assuming $q = 1$, equation (6) can be explicitly solved for $\tau^*$ yielding:

$$\tau^* = \frac{\eta \beta_h r - (r+\delta)(1-\chi_h)}{\eta \beta_h r \chi_h} \quad (7)$$

\(^{14}\)As Sørensen (2016) points out, to define the undistorted or social cost of capital we should include a risk premium that depends on $\beta$. Because we assume that $\beta$ is exogenous, without loss of generality we can define $r = r_0 + p_S(\beta)$.
Any positive profit corporate tax different from \( \tau^* \) implies a cost of capital which is different from \((r + \delta)\), in turn implying that such tax rate is not neutral.

**Corollary 1.** If Condition A holds and \( \chi_h \leq 1 \) and \( \beta_h \leq 1 \), the neutral tax rate \((\tau^*)\) is strictly positive and not greater than 1.

**Proof.** From condition A,

\[
1 - \chi_h < \frac{\eta \beta_h r}{r + \delta}
\]

Replacing this in (7), we get that \( \tau^* > 0 \). To prove that \( \tau^* \) is less or equal to 1, first notice that \( \tau^* \) increases with \( \beta_h \) and \( \chi_h \). Thus we have that:

\[
\tau^* = \frac{\eta \beta_h r - (r + \delta)(1 - \chi_h)}{\eta \beta_h r \chi_h} \leq \frac{1}{\chi_h} - \frac{(r + \delta)(1 - \chi_h)}{\eta r \chi_h} \leq 1. \]

It is important to emphasize that Condition A is not a curiosity; in fact, it is satisfied by many tax codes of countries that use tax allowances for capital depreciation and for interest costs (Mirless et al., 2011; Klemm and Van Parys, 2012; Abbas and Klemm, 2013; Hanappi, 2018).\(^{15}\)

If condition A does not hold then there is no positive tax rate that allows for weak neutrality and thus for a tax paradox.\(^{16}\)

Moreover, the following result follows.

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\(^{15}\) Taking values from Hannapy (2018) which contains values for net present value of capital allowances (our \( \chi_h \)), economic deprecations (\( \delta \)) and interest rates, we show that condition A holds in all OCDE countries for power generation assets, manufacturing and scientific R&D.

\(^{16}\) In the US for example, the 2017 tax reform made \( \chi_h = 1 \) and the level of \( \eta \) is made to depend on the business-adjusted taxable income plus floor plan financing interest. So, for firms having \( \eta \) greater than zero we have that condition A may be satisfied.
**Result 1.** It is conceivable that a firm facing a low but positive corporate tax rate is affected by greater inefficiency or deadweight loss and invest less than an identical firm facing a higher tax rate.

**Proof:** Using (6) and (7) it follows that the minimum cost of capital occurs if $\tau^c = 1 - \sqrt{1 - \tau^*}$, which implies that $\tau^* > \tau^c$. Then, if $\tau_1$ is less than $\tau^c$, the cost of capital should be higher with $\tau_1$ than $\tau^c$. \[\blacksquare\]

Also, we then have the following corollary to Proposition 1,

**Corollary 2.** The level of the user cost of capital is lower (higher) than its undistorted level $(r + \delta)$ if $0 < \tau < \tau^*(\tau > \tau^*)$ and is equal to its undistorted level if $\tau = \tau^*$. Moreover, the user cost of capital is decreasing (increasing) in the tax rate within the $0 - \tau^c (\tau^c - \tau^*)$ interval and increasing thereafter. That is, the user cost of capital exhibits a U-shaped relationship with the corporate tax rate.

**Proof.** Follows directly from Proposition 1.

Corollary 2 shows that deadweight losses may occur if either the after-tax user cost of capital is below or above its undistorted level ($\varphi^*$), which in turn implies that such losses occur if the level of investment is above or below its socially optimal level, respectively. When the after-tax user cost of capital is below $\varphi^*$ it implies that the tax system effectively subsidizes the cost of capital to a level below the social cost and hence it induces firms to implement investments with a rate of return below the socially optimal level. When the tax rate is within the interval $0 < \tau < \tau^*$, we have that there are deadweight losses which reach
their maximum when $\tau = \tau^c$. That is, in this interval there exists a tax paradox: an increase of the corporate tax rate increases investment.

From the Corollary 2 it follows that if the marginal product of capital is decreasing then the level of capital investment is higher (lower) than its neutral or optimal level if $0 < \tau < \tau^*(\tau > \tau^*)$ and it is optimal or neutral if either $\tau = \tau^*$ or if $\tau = 0$. As shown in Figure 1, the optimal (neutral) tax rate, $\tau^*$, is on the upward sloping side of the (U-shaped) user cost function, $c = c(\tau)$. The upper panel of the figure shows the U-shaped relationship between the corporate tax and the user cost of capital. The lower panel exhibits the inverted U-shaped relationship between the corporate tax rate and capital investment.

As can be seen in Figure 1, the level of capital investment is socially optimal if $\tau$ is either zero or equal to $\tau^*$. As $\tau$ rises from zero to $\tau^c$ capital increases to levels above its social optimum and hence efficiency falls. If after reaching its $\tau^c$ value, at which the user cost of capital attains its minimum, if $\tau$ continues to increase towards $\tau^*$, capital falls towards its optimal level and hence efficiency improves together with a falling level of capital. As $\tau$ rises above $\tau^*$investment, capital stock and efficiency decline with respect to their social optimum levels at $\tau^*$.

An implication of the above analysis is that it is possible that if condition A is satisfied an increase in the corporate tax rate may not reduce investment. Empirically, there are studies that tend to corroborate this possibility, including Jorgenson and Hall (1971) and Ljungqvist and Smolyansky (1990) for the US, Stanford (2011) for Canada and Bustos et.al (2004) for
Chile, which show that at least for certain types of firms raising taxes do not lower investment.

Figure 1. Non-monotonic relationship between the tax rate and the user cost of capital and capital investment

Result 2: On the connection between weak tax neutrality and the tax paradox.

Corollary 2 shows the connection between weak neutrality and the concept of tax paradox; in the interval \((0, \tau^c)\) the cost of capital decreases with the corporate tax rate which in turn causes an increase of the firm’s investment. However, the cost of capital increases in the
[τ, τ] interval. That is, weak neutrality is only achievable if the initial tax paradox is more than offset at higher values of the corporate tax; the U-shaped relationship between the cost of capital and the corporate tax is a necessary condition for weak neutrality. Without this U-shaped relationship a tax paradox may exist, but weak neutrality might not be feasible. That is, the existence of a tax paradox is a necessary but not sufficient condition for weak neutrality.

Returning to the differences in the definition of weak neutrality between our approach and that of Gries et al. (2012), we can see that when the marginal effect of the tax is zero (which occurs when the tax equals τ), the resulting tax rate is not neutral. Also, neutrality attains at a tax rate where the marginal effect of the tax on the user cost of capital is positive, not in general zero.

3. On the intuition of weak neutrality

We now use an elementary model to provide some intuition to our results, illustrating the effects on the firms’ optimal capital (and investment) of two identical increases of the corporate tax rate, one occurring when the tax rate is high and the other when it is low. The first order condition for profit maximization implies that the firm equalizes the value of the after-tax marginal product of capital to their after-tax marginal cost. Define \( \tilde{c}(\tau) \equiv \left[1 - \tau \gamma h \right] (r(1 - \eta \beta \tau) + \delta) \), where \( \tilde{c}(\tau) \) is decreasing and strictly convex in \( \tau \).\(^{17}\) Then the first order condition for profit maximization is \( (1 - \tau)F'(K) = \tilde{c}(\tau) \), where \( F'(K) \) is the

\(^{17}\) Of course, from equation (3) it follows that \( \tilde{c}(\tau) \) is related to the user cost of capital as follows: \( c(\tau) \equiv \frac{\tilde{c}(\tau)}{(1 - \tau)} \)
marginal product of capital ($K$). Thus, an increase of $\tau$ reduces the after-tax marginal product of capital but it also reduces the after-tax cost, $\tilde{c}(\tau)$. Therefore, the net effect of $\tau$ on capital is ambiguous depending on the relative strength of these two opposite effects.

Figure 2 illustrates this. It shows the effects of alternative tax rates, $\tau^1 < \tau^2 < \tau^3 < \tau^4$, with $\tau^2 - \tau^1 = \tau^4 - \tau^3$. The top panel shows the case where the corporate tax rates $\tau^1$ and $\tau^2$ are low enough allowing capital investment to rise in response to a higher tax and the bottom panel shows the case where the corporate tax rates $\tau^3$ and $\tau^4$ are high enough causing capital investment to decline in response to a tax rate increase. The key issue is that while the shift of the $(1 - \tau)F'(K)$ schedule is equal for any identical tax change, $\tilde{c}(\tau)$ being strictly convex and decreasing in $\tau$, falls less in response to an identical tax rise when taxes are high than when they are initially low.

This is shown in Figure 2 by the fact that the downward shift of $\tilde{c}(\tau)$ is smaller in the bottom panel which is associated with high tax rates than in the top panel which reflects the situation with lower tax rates. The case depicted in the top panel of the figure illustrates an increase of firm’s optimal capital (and a higher desired investment) as a result of a higher tax, while the bottom panel shows the case when an identical tax hike causes a fall in optimal capital (and lower investment). Noting that given continuity of the functions involved, there must be a tax level at which both effects exactly off set each other, a U-shaped relationship between the corporate tax and capital follows.
3. Weak neutrality: Real world pertinence

We discuss certain key issues related to the practical policy implications of weak neutrality as well as an issue generally considered of interest in the tax neutrality literature: The implications of asset heterogeneity.

4.1. Weak neutrality and the maximum tax-free rate of return to capital

In strongly neutral tax systems the maximum tax-free rate of return to capital ($MRR$) is equal to the opportunity cost of capital. That is, the taxable profit rate in these systems is
equal to the excess profit rate above the $MRR = r$. In this section we show that weakly neutral tax systems also allow a tax exempt $MRR$ although this rate is not in general equal to $r$. Moreover, unlike the case of strongly neutral systems, the $MRR$ is not fixed but rather dependent on the corporate tax rate itself. We show that the $MRR$ in weakly neutral systems could even be higher than that of strongly neutral systems.

**Proposition 2.** The minimum taxable profit rate of strongly or weakly neutral tax systems is equal to the excess of the actual profit rate over the $MRR$, where

$$MRR = \eta \beta_h r (1 - \tau \chi_h) + (r + \delta) \chi_h - \delta$$

(8)

If $\chi_h = 1$ and $\eta = 1$ then $MRR = r$, the system is strongly neutral as it is in the case of the Cash Flow method (CFM). If $0 < \chi_h < 1, \eta > 0$ (debt interest costs are at least partially tax-deductible) and condition A is satisfied, then the system is weakly neutral and the $MRR$ is decreasing in the tax rate.

**Proof.** Consider the profit maximization of a firm affected by a tax, $\tau$,

$$\max_{K} \{(1 - \tau)F(K) - [(1 - \tau \chi_h)\big[(1 - \eta \beta_h \tau)r + \delta\big]qK\}$$

(9)

Without loss of generality, by using appropriate units, we can assume that $q = 1$. We rewrite (15) as,

$$\max_{K} \{F(K) - (r + \delta)K - \tau[F(K) - \beta_h \eta r (1 - \chi_h \tau)K - (r + \delta)\chi_h K]\}$$

(10)

The expression in square brackets in (10) is the tax base or taxable profits. If the tax base is less than or equal to zero, then the firm pays no tax. Thus, the firm is tax-exempted if
Inequality (11) can be rewritten as,

\[ F(K) - \delta K - \{(1 - \chi_h \tau)\eta \beta_h r + (r + \delta)\chi_h - \delta \} K \leq 0. \quad (12) \]

Define the profit rate as \( \pi/K \equiv [F(K) - \delta K]/K. \) Thus, from (12) it follows that the tax base is less or equal to zero if,

\[ MRR = (1 - \chi_h \tau)\eta \beta_h r + (r + \delta)\chi_h - \delta \quad (13) \]

If \( \chi_i = 1 \) and \( \beta = 0 \) then \( MRR = r. \)

The optimal tax that maximizes tax revenues without distorting the user cost of capital is the one that makes \( MRR=r. \) Using (13) we obtain that the optimal tax, \( \tau^* \), is defined by,

\[ (1 - \chi_h \tau^*)\eta \beta_h r + (r + \delta)\chi_h - \delta = r \quad (14) \]

Solving (14) we obtain that the optimal tax is

\[ \tau_h^* = \frac{\eta \beta_h r - (r + \delta)(1 - \chi)}{\eta \beta_h r \chi_h} \quad (15) \]

When \( \tau = \tau_h^* \) the system is neutral as the taxable base is equal to the economic rents. If \( \tau < \tau_h^* \) then \( MRR> r \) the taxable base is less than the economic rents meaning that the system allows the firm to avoid paying taxes even if it obtains some economic rents; and, if \( \tau > \tau_h^* \) then \( MRR< r \) meaning that the firm will pay taxes on a higher level of income than their economic rents. That is, in this case the tax-free rate of return is below the opportunity cost of capital and therefore the tax distorts investments. ■
Proposition 2 shows that in both systems firms do not pay taxes on all their profits. In this sense the corporate tax system is like the personal tax system where individuals pay taxes according to brackets. However, in the case of the weakly neutral corporate tax system there are only two tax brackets, a tax-exempt one that applies to firms with profits less or equal to the MRR and a taxable bracket that applies to firms obtaining profits above the MRR. Also, as in the personal tax case, the corporate tax applies to the difference between the firm’s total profit rate and the MRR profit, not to its total profits.

Another important implication of Proposition 2 is that unlike the personal income tax case, the tax boundary (the MRR) in the weakly neutral system is affected by the corporate tax rate. This means that, for example, as the legal tax rate increases from $\tau^c$ towards $\tau^*$ the maximum tax-exempt rate of return to capital falls and, therefore, the taxable base increases. This in turn implies that tax revenues may increase more than proportionally to the increase of the tax rate, while at the same time inefficiency is reduced because the user cost of capital is brought closer to its undistorted level.

**REMARK:** The tax base used by the strongly neutral (SN) systems is equal to the pure economic rent ($F(K) - (r + \delta)K$), which is the reason why it is possible to apply any corporate tax rate (even 100%) without causing distortion or inefficiency. In the weakly neutral (WN) system, the tax base is generally dependent on the actual tax rate and is different to the pure rent unless the tax rate is equal to the optimal one ($\tau = \tau^*$). *When the tax rate is optimal the tax base becomes equal to the pure profit and for this reason the positive profit corporate tax causes no distortion or inefficiency.* Also, the tax revenues of the two systems are identical if the tax rate chosen when using a SN system is equal to the
optimal tax rate under the WN system. The only difference between the two systems is that the SN system may capture up to the whole economic rents without causing any distortion while the WN system cannot in general capture the entire economic rents if the optimal tax rate is different from $\tau^*$. From a practical point of view this result is important for the evaluation of tax reform. A concern often raised about increasing corporate taxes is that it may negatively affect small firms that usually have lower rates of return to capital than larger firms. However, from the previous analysis it follows that if a (small) firm has initially a rate of return to capital below the MRR it may not be affected by the tax increase as long as the post-tax reform MRR is still above the firm`s rate of return to capital. For example, suppose that prior to the reform the $MRR = 12\%$, then a firm with a pre-tax reform rate of return of 8% will continue to pay no taxes after the tax increase if the tax rise causes the MRR to decrease to a rate still above 8%.

4.2 The multi asset case: Weak neutrality and the neutrality loci. In order to consider the case of multi asset heterogeneity we first show a dual of the result regarding Proposition 1: if condition A is satisfied then, any positive corporate tax can be rendered non-distortive by adjusting at least one non-tax rate parameter of the tax system. Consider an arbitrary tax $0 < \bar{\tau} < 1$. Then define all the combinations of $\chi_j$ and $\eta$ satisfying $\chi_j > \frac{(1-\eta \beta_h)r+\delta}{r+\delta}$ that solve the following neutrality equation,

$$c(\bar{\tau} ; \chi_j, \eta \beta_h) = r + \delta$$

(16)
Using (5) and (8) we obtain,

\[ x_j^* = \frac{(r+\delta)-\eta \beta_h r}{(r+\delta)-\overline{\tau} \eta \beta_h r} \]  \hspace{1cm} (17)

Equation (17) allows us to analytically determine the combinations of \( x_j \) and \( \eta \beta_h \) under which \( \overline{\tau} \) is neutral. For example, when \( \eta = 0 \) and \( x_j = 1 \) the tax system would be globally neutral. In general, however, if \( \eta \beta_h > 0 \) the level of \( x_j \) that solves (17) is a function of \( \eta \); it follows that the slope of (17) is negative \((\partial x_j / \partial \eta \beta_h<0)\).

Figure 2 shows three loci of weak neutrality for three different values of \( \tau \), \( \tau^0 < \tau^1 < \tau^2 \); for each value of \( \tau \), the locus of weak neutrality is obtained by the combinations of \( x \) and \( \eta \beta_h \) that make \( \tau \) non-distortive. These loci are all above the grid area of Figure 3 which depicts the area under which Condition A is satisfied, that is when \( x_j > \frac{(1-\eta \beta_h) r+\delta}{r+\delta} \).

Figure 3. Three loci for which the corporate income tax is weak neutral
Also, notice that,

$$\frac{\partial \chi_j}{\partial \tau} = \eta \beta_h r (r + \delta_j - \eta \beta_h r) \left( (r + \delta_j) - \tau \eta \beta_h r \right)^2 > 0$$

That is, an increase in the tax rate will require a higher depreciation allowance to achieve investment neutrality.

### 4.3 Weak neutrality and asset heterogeneity

The recent literature on taxation and investment (Zwick and Mahon, 2017; Fatica, 2017a) finds that corporate taxes have different effects on different assets. Tax codes normally provide for capital depreciation rules ($\chi_j$) that are asset specific (Clarke, 1993). It is also well-known that the tax distortions to investment are higher the larger the gap between tax depreciation and economic depreciation (Fatica, 2017b). This has important implications for the analysis of weak neutrality in the context of asset heterogeneity.

Recall equation (3) which now should hold for each asset $j$ and for each firm $h$ characterized by a debt ratio, $\beta_h$. Thus, the capital arbitrage condition (3) becomes,

$$q_{hj}(t) = \int_t^\infty \left[ (1 - \tau) c_{hj}(s) e^{-(\delta_j + i_h)(s-t)} \right] ds + \tau \chi_j q_{hj}(t) \tag{3'}$$

Where,

$$i_h = (1 - \eta \beta_h \tau) r$$

Differentiating (3’) with respect to time, we have,

$$c(t)_{hj} = \frac{\left( (1 - \eta \beta_h \tau r + \delta_j) [1 - \chi_j \tau] \right)}{(1 - \tau)}$$
A tax rate \( \bar{\tau} \) is weakly neutrality if and only if,

\[
c(t)_{hj} = \frac{\left(1 - \eta \beta_h \bar{\tau} + \delta_j\right)[1 - \chi_{hj} \bar{\tau}]}{(1 - \bar{\tau})} = r + \delta_j, \quad j = 1, \ldots, J; h = 1, \ldots, H
\]  

(18)

Therefore, weak tax neutrality requires that the government set one unique \( \chi_{hj} \) for each asset type \( j \) and firm debt type, \( \beta_h \). Tax codes routinely allow for many different depreciation allowances for the large number of asset types that they considered. So, the only difference with the standard approach is that achieving weak tax neutrality requires to apply an “optimal” tax depreciation allowance for each of the \( J \) assets instead of a more or less arbitrary ones as is the common practice.

However, there is an additional complication associated with the fact that firms have different levels of debt which given the fact that debt interest are often tax deductible implies that the effective interest rate is different for each type of firm debt ratio. Assume that we can approximate the space of firms’ debt ratios into \( H \) types of discrete subspaces. Then there are \( JxH \) equations (18) which can be solved separately for each \( \chi_{hj} \).\(^{18}\) This implies that that weak tax neutrality requires that the tax law specify \( JxH \) asset optimal depreciation allowances to account for the full heterogeneity of assets and firms’ debt. Weak neutrality would thus be accomplished, or equivalently, if we allow for \( JxH \) asset depreciation allowances (\( \chi_{hj} \)) the tax level \( \bar{\tau} \) is rendered neutral. From equation (18) it follows that given \( \beta_h, \bar{\tau} \) and \( r \) fixed, if condition A holds for each asset \( j \), it is possible to define a depreciation allowance schedule \( \chi_{hj}(\bar{\tau}, \beta_h, \delta_j, r) \equiv \chi_{hj}(\delta_j) \).

\(^{18}\) The solution is obtained for each equation since there is only one endogenous variable to be obtained from each of the \( JxH \) equations, \( \chi_{hj} \).
We notice that:

\[
\frac{\partial \chi_{hj}}{\partial \delta_j} = \frac{\eta \beta_h (1-\bar{r})}{(r+\delta_j-\bar{r} \eta \beta_h r)^2} > 0 \quad \text{and} \quad \frac{\partial^2 \chi_{hj}}{\partial \delta_j^2} = -\frac{2 \eta \beta_h (1-\bar{r})}{(r+\delta_j-\bar{r} \eta \beta_h r)^2} < 0
\]

That is, the “neutral” depreciation allowance for asset $j$ is an increasing and concave function of the true depreciation of asset $j$.

Figure 4: “Neutral” tax depreciation allowance in firm $h$ for asset $j$, $\chi_{hj}$, as a function of the true depreciation $\delta_j$.

4.4 Interest rates.

The market interest rate affects the condition for weak neutrality as well as the size of the depreciation-neutral allowances. It is particularly relevant to know what happens if the
interest rate is close to 0 (the zero-lower bound) given the current economic conditions characterized by historically low interest rates. In this case condition A becomes \( X_{hj} > 1 \) for each asset and firm. That is, there is no weak neutrality. The intuition for this is that an interest rate equal to 0 means no interest payment deduction from the tax base; thus the only mechanism to compensate a higher tax rate is the depreciation allowance. However, while zero rates have at recent times prevailed for the prime rate, most of the firms have not enjoyed zero interest rates. Even in an environment of extremely lax monetary policy like in recent years firms do pay positive interest rates on their debt.

Also, a higher interest rate means that condition A is more likely to hold. This is because the higher the interest rate, the higher the value for the firm of the tax allowance due to interest deduction. Moreover, a higher interest rate means a lower ‘neutral’ depreciation allowance deduction. To see this, notice that:

\[
\frac{\partial \chi_{hj}}{\partial r} = \frac{-\eta \beta_h \delta_j (1 - \tau)}{\left((r + \delta_j) - \tau \eta \beta_h r\right)^2} < 0
\]

### 4.5 Firms financial policy depending on \( \tau \).

So far, we have assumed that the firms’ debt ratios are exogenous. However, empirical evidence suggests that a firm’s financial policy is in part endogenous depending on the level of the corporate tax. An increase of the tax rate generates incentives to use debt as a financial instrument because there are more interest payment deductions from the tax base. Thus, it is intuitive to assume that the debt-to-asset ratio, \( \beta(\tau) \), is an increasing function of \( \tau \).
We first consider the simplest case in which the debt-to-asset ratio is proportional to the tax rate, \( \beta(\tau) = \beta_1 \tau \), where \( \beta_1 > 0 \). In this case the condition represented by Equation (6) changes slightly,

\[
[1 - \chi \tau^*] \frac{(r(1-\eta \beta_1 \tau^2)+\delta)}{(1-\tau^*)} = r + \delta
\]  

(6’)

Weak neutrality requires the existence of a solution to Equation (6’) with \( 0 < \tau^* < 1 \) and \( \chi < 1 \). Equation (6’) is now quadratic in \( \tau^* \) and its solution is,

\[
\tau^* = \frac{1 \pm \sqrt{1-4\chi D}}{2\chi},
\]  

(7’)

Where \( D \equiv \frac{(1-\chi)(r+\delta)}{\beta_1 r} > 0 \) for \( \chi \leq 1 \). Existence of weak neutrality requires that this solution be real and \( 0 < \tau^* < 1 \). The solution is real if \( 4\chi D < 1 \). That is, if and only if,

**Condition A’**

\[
\chi(1-\chi) < \frac{\beta_1 r}{4(r+\delta)}
\]

Since \( \chi(1-\chi) \leq 1/4 \), condition A’ is feasible as long as \( (1-\beta_1)r + \delta \geq 0 \). This is a weak condition which always hold if \( \beta_1 \) is not much greater than 1 (that is, if the debt rate is not too much greater than the tax rate).

Thus, we have that there exist two neutral tax rates that satisfy the condition that \( 0 < \tau^* < 1 \): (a) A low tax rate,

\[
\tau_1^* = \frac{1 - \sqrt{1-4\chi D}}{2\chi}
\]

And (b) a high tax rate,
\[ \tau_2^* = \frac{1 + \sqrt{1 - 4 \chi D}}{2 \chi}, \]

We note that, given Condition A’, both \( \tau_1^* \) and \( \tau_2^* \) are positive. Also, \( \tau_1^* < 1 \) and \( \tau_2^* \) is less than 1 if \( \beta_1 r < r + \delta \) (which is the same condition required for Condition A’ to be feasible). That is, the high tax rate is less than one as long as \( \beta_1 \) is not much greater than 1.

Neutrality in the case of the high tax solution requires a high depreciation allowance requiring that \( \chi > \frac{1}{2} \) while neutrality in the low tax rate can be achieved even if \( \chi < \frac{1}{2} \). This is of course highly intuitive.

Thus, the new condition A’ is somehow more complicated than the original condition A but it is empirically verifiable and certainly feasible. Also, condition A’ assures the existence of two neutral tax rates that are positive and less than one. That is, weak neutrality still holds in this case.

Next we consider the more general case in which \( \beta(\tau) \) is increasing in \( \tau \) but not necessarily proportional. First, let’s use a second order Taylor expansion for \( \beta(\tau) \) around \( \tau = 0 \). Then we have:

\[ \beta_h(\tau) = \beta_h^0 + \beta_h^1 \tau + \beta_h^2 \tau^2 \]

We can interpret the coefficient \( \beta_h^0 \) as the debt ratio that would prevail if firms faced a tax rate equal to zero. Under the plausible assumption that \( \beta_h(\tau) \) is increasing in \( \tau \), the sum of the remaining terms, \( \beta_h^1 \tau + \beta_h^2 \tau^2 \), is expected to be positive. Given this, the condition A for weak neutrality evaluated at \( \tau = 0 \) changes to:
Condition A''

\[ \chi_{hj} > \frac{(1-\eta\beta_h) r + \delta}{r + \delta} \]

The new condition A'' requires that the firm would still borrow even if the tax rate were zero. This means that there are reasons other than taxes for firms to borrow in the capital market. That is, instead of depending on the average debt, it depends on the undistorted financial policy (policy without taxes).

Another important implication for the analysis is that an increase in the tax rate will increase the depreciation rate required for neutrality more than in the case when \( \beta \) is exogenous. To see this notice that:

\[
\frac{\partial \chi_{hj}}{\partial \tau} = \frac{\eta \beta_h r (r + \delta_i - \eta \beta_h r) - (1-\tau)(r + \delta_i) \eta \beta_h' r}{((r + \delta_i) - \tau \eta \beta_h r)^2} < \frac{\eta \beta_h r (r + \delta_i - \eta \beta_h r)}{(r + \delta_i) - \tau \eta \beta_h r)^2}
\]

This implies that an increase in taxes will be compensated by an increase in the financial policy, making the depreciation allowance effect stronger. Thus, a lower depreciation allowance will be required to achieve investment neutrality.

4. Policy applications of weak neutrality.

The most important insight from this model is that it is possible to limit the tax benefit for the accelerated depreciation given a corporate tax rate and the interest payment allowances. As indicated earlier, this could be possible by solving the \( JxH \) equations (18) above for the \( JxH \) levels of “optimal” asset depreciation allowances.

Given a corporate tax rate, interest rate and other parameters that are common to all firms and asset types, the above equations can independently be solved for each type of asset
and firm debt for optimal depreciation allowances \( (\chi_{hj}) \). Most tax codes consider a limited number of asset types to determine depreciation allowances specific to each of them. Suppose the tax code considers \( J \) types of assets. In addition, one may divide the space of firms’ debt ratios between 0 and the maximum observed rate to approximate the whole spectrum of observed debt ratios through a discrete number of spaces. Let’s say we define \( H \) levels of debt ratios. Then there are \( J \times H \) combinations of depreciation allowances which satisfy the \( J \times H \) equations (18). If for example the tax code considers 3 types of assets (assets with short, medium and long depreciation periods) and we can divide the firm’s debt space into 3 types as well (low, medium and high debt ratios) then there will be 9 equations (18) that must be independently solved for each of the 9 asset depreciation allowances. This will render a prevailing corporate tax rate neutral. Of course, in reality it might be necessary to specify a much greater number of asset and firm types to obtain a more adequate approximation to tax neutrality. But computing even a much larger number of allowance combinations should be no obstacle given the current computing capabilities.

6. Discussion of the US tax reform

On December 22, 2017, it was signed the law H.R. 1, originally known as the Tax Cuts and Jobs Act. This reform reduces the taxes paid by individuals and corporations. Particularly there were three major changes for the corporate sector. First was a reduction of the statutory corporate tax rate from 35% to 21%. Second, there was an increase of the bonus for immediate depreciation from 50% to 100% for qualified property acquired and placed
in service after Sept. 27, 2017, and before Jan. 1, 2023.\textsuperscript{19} Finally, the new law imposes a limitation on the deduction of net business interest expense to a maximum of 30% of the business’s adjusted taxable income.\textsuperscript{20}

Using the framework developed in this paper, the tax reform would cause $\chi_{hj} = 1$, $\tau = 0.21$ and a reduction of the interest payment allowed by law, $\eta$. The fact that $\chi_{hj} = 1$ implies that Condition A is likely to be satisfied. Hence, weak tax neutrality may exist. Moreover, increasing the value of $\chi_{hj}$ to one while maintaining interest deductions it is possible that the resulting after-tax user cost of capital, especially for firms having a large level of adjusted taxable income, to become less than the true or economic user cost of capital.

7. Conclusion

We have introduced the concept of weak neutrality of a tax system as an application of the concept of ‘tax paradox’ which, under certain conditions elucidated in this paper, allows us to derive an optimal positive corporate profit tax that is neutral. We have clarified the difference between strongly neutral and weakly neutral systems. We developed the conditions under which weak neutrality exists, that is when there exist at least one positive

\textsuperscript{19} The bonus depreciation percentage for qualified property that a taxpayer acquired before Sept. 28, 2017, and placed in service before Jan. 1, 2018, remains at 50 percent. Special rules apply for longer production period property and certain aircraft. The definition of property eligible for 100 percent bonus depreciation was expanded to include used qualified property acquired and placed in service after Sept. 27, 2017. Also, after 2023, the immediate bonus depreciation will decline by 20% each year.

\textsuperscript{20} The conference report’s explanatory statement indicates that the section 163(j) limitation should be applied after other interest disallowance, deferral, capitalization or other limitation provisions. Thus, the provision would apply to interest the deduction for which has been deferred to a later tax year under some other provision. The provision applies to all businesses, regardless of form, and any disallowance or excess limitation would generally be determined at the filer level (e.g., at the partnership level instead of the partner level).
corporate tax rate that allows the user cost of capital to be equal before and after tax. Thus, even if strong neutrality is not feasible we have shown that under certain plausible conditions, weak tax neutrality is feasible.

An important insight arising from this paper is that under the usual tax allowances prevailing in many countries, the after-tax capital user cost curve may be non-monotonic in the corporate tax rate, having a U-shaped. While the possibility of a non-monotonic relationship between the tax rate and the user cost of capital is well known in the literature, a specific U-shaped relationship which arises under plausible empirical conditions has not been recognized. It is within the context of such U-shaped relationship that the concept of weak neutrality arises. This is a new result with potentially important public policy connotations.

We examined the practical value of the concept of the weak tax neutrality in the more real world case where assets and firms’ debt ratios are heterogenous. We showed that despite this heterogeneity this concept can have high empirical relevance. We have shown that, under the conditions that allow for the existence of tax neutrality, one can make any positive tax rate to be neutral by choosing the right combination of asset depreciation allowances. The fact that most tax codes do allow for different asset depreciation allowances greatly increases the practical feasibility of our proposal. The only thing that is required is that the levels of the asset depreciation allowances must be set optimally. Furthermore, we have shown that the optimal depreciation allowances should be different not only for each asset but should also consider the level of debt ratio of firms. An approximation to the optimal asset depreciation is obtained by considering a discrete
number of debt ratio intervals (let’s say high, medium and low debt ratios). Of course, the
greater the number of debt ratios intervals and assets considered the better would be the
approximation. The optimal allowances can be obtained by independently solving simple
equations.
References


Appendix

Consider the following generalization of the capital arbitrage condition proposed originally by Hall and Jorgenson (1967),

\[ q(t) = \int_t^\infty [(1 - \tau)c(s)e^{-(\delta + i)(s-t)}]ds + \tau \chi_i q(t) \]

Rearranging terms we have:

\[ q(t)(1 - \tau \chi_i) = \int_t^\infty [(1 - \tau)c(s)e^{-(\delta + i)(s-t)}]ds \] (A1)

To derive equation (5) we need to differentiate (A1) with respect to \( t \) and equalizing it to 0.

Assuming \( \frac{dq(t)}{dt} = 0 \) and using the Leibnitz rule, we have:

\[ 0 = -(1 - \tau)c(t) + (\delta + i_i) \int_t^\infty [(1 - \tau)c(s)e^{-(\delta + i)(s-t)}]ds \]

Now, using (A1), we have that:

\[ c(t) = \frac{q(t)(r(1 - \eta \beta_i) + \delta)[1 - \tau \chi_i]}{(1 - \tau)} \]

Which is equation (5).